Math 3120-001: Practice Test One

Due on June 8, 2011

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Your Name:.....

Problem 1

Classify each of the following as an Ordinary Differential Equation or a partial differential equation, give the order, and indicate the dependent and independent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or non-linear, autonomous or not autonomous.

(a)

$$\frac{dy}{dx} = \frac{2xy}{y-1}.$$

(b)

$$\frac{dP}{dt} = \frac{7}{100}P.$$

(c)

$$\frac{\partial^2 u}{\partial t^2} - 16\frac{\partial^2 u}{\partial x^2} = 0.$$

Problem 2

Verify that

$$y = \tan(x^3 + C)$$

satisfies the differential equation

$$\frac{dy}{dx} = 3x^2(y^2 + 1).$$

Find the values of C so that $y(0) = 1$.

Problem 3

Solve the the following equations:

(a)

$$\frac{dy}{dx} + 3xy = 0$$

(b)

$$2\sqrt{x}\frac{dy}{dx} = \sqrt{1-y^2}.$$

Problem 4

If $P(t)$ is the amount of dollars in a savings bank account that pays a yearly interest rate of $r\%$ compounded continuously, then

$$\frac{dP}{dt} = \frac{r}{100}P,$$

where t is measured in years. Assume the interest is 5% annually, $P(0) = \$1000$, and no monies are withdrawn.

(a) Solve the differential equation.

(b) How much will be in the account after 2 years?

(c) When will the account reach \$4000?

Problem 5

Determine for which values of m the function $\phi(x) = x^m$ is a solution to the differential equation

$$3x^2\frac{d^2y}{dx^2} + 11x\frac{dy}{dx} - 3y = 0.$$

Problem 6

Determine whether the given relation

$$y - \ln(y) = x^2 + 1$$

is an implicit solution to the

$$\frac{dy}{dx} = \frac{2xy}{y-1}.$$

Problem 7

Consider the differential equation

$$\frac{dP}{dt} = P(P-1)(2-P)$$

for the population P (in thousand) of a certain species. Can a population of 900 ever increase to 1200? Explain your answer.

Problem 8

Use Euler's method with step size $h = 0.5$ to approximate the solution to the initial value problem

$$y' = \frac{1}{x}(y^2 + y), \;\; y(1) = 1.$$

at the points $x = 1.5$ and $x = 2$.

Problem 9

Determine whether equation

$$\frac{dx}{dt} + xt = e^x$$

is separable or not separable.