

Math 3120-001: Practice Test One

August, 2011

Professor Z. Sinkala

Your Name:.....

Problem 1

Classify each of the following as an Ordinary Differential Equation or a partial differential equation, give the order, and indicate the dependent and independent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or non-linear, autonomous or not autonomous.

(a)

$$\left(\frac{dy}{dx}\right)^2 = \frac{2x^2y}{x-1}.$$

(b)

$$\frac{d^2P}{dt^2} = \frac{7}{100}P^2.$$

(c)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{1}{x} \frac{\partial u}{\partial x} + 3u.$$

Problem 2

Verify that

$$y = \tan(x^3 + C)$$

satisfies the differential equation

$$\frac{dy}{dx} = 3x^2(y^2 + 1).$$

Find the values of C so that $y(0) = 1$.

Problem 3

Solve the the following equations:

(a)

$$\frac{dy}{dx} + 3xy = 0$$

(b)

$$\frac{dy}{dx} = 8x^3 e^{-2y}.$$

Problem 4

According to Newton's Law of Cooling if an object at temperature T is immersed in a medium having constant temperature M , then the rate of change of T is proportional to the difference of temperature $M - T$. This gives the differential equation

$$\frac{dT}{dt} = k(M - T) :$$

A thermometer reading 100 degrees is placed in a medium having constant temperature of 70 degrees. After 6 minutes the thermometer reads 80 degrees. What is the reading after 20 minutes?

- (a) After 5 minutes the thermometer reads 80 degrees.
- (b) What is the reading after 10 minutes?
- (c) What is the reading as time approaches infinity?

Problem 5

Determine whether the function

$$x = \cos(2t)$$

is a solution to the equation

$$\frac{dx}{dt} + tx = \sin(2t).$$

Problem 6

Determine whether the given relation

$$x^2 + y^2 = 1$$

is an implicit solution to the

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Problem 7

Consider the differential equation

$$\frac{dP}{dt} = P(P - 100)(P - 200)$$

for the population P (in thousand) of a certain species.

- (a) Can a population of 80 ever increase to 300?
- (b) Can a population of 80 ever increase to 180?
- (c) Can a population of 80 ever decline to 18?
- (c) If the initial population is 190, what can you say about the limit population $\lim_{t \rightarrow +\infty} P(t)$?

Problem 8

(a) Determine whether equation

$$\frac{dx}{dt} + xt = x$$

is separable, linear or both.

(b) Determine whether equation

$$\frac{dy}{dx} + \frac{3}{x}y = 0$$

is separable, linear or both.

Problem 9

Find the general solution to the equation

$$\frac{dy}{dx} - \frac{y}{x} = 2x + 1.$$