

Math 3120-001: Practice Test four

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Your Name:.....

Problem 1

Find the critical point set for the following system

$$\begin{cases} \frac{dx}{dt} = x - y \\ \frac{dy}{dt} = x^2 + y^2 - 1 \end{cases}$$

Problem 2

Find the critical point set for the following system

$$\begin{cases} \frac{dx}{dt} = y - 1 \\ \frac{dy}{dt} = x + y + 5 \end{cases}$$

Problem 3

Solve the phase plane for the following system

$$\begin{cases} \frac{dx}{dt} = 2y \\ \frac{dy}{dt} = 2x \end{cases}$$

Problem 4

Verify that the pair $x(t) = t + 1$, $y(t) = t^3 + 3t^2 + 3t$ is a solution to the following

$$\begin{cases} \frac{dx}{dt} = 1 \\ \frac{dy}{dt} = 3x^2 \end{cases}$$

Problem 5

Solve the phase plane for the following system

$$\begin{cases} \frac{dx}{dt} = -8y \\ \frac{dy}{dt} = 18x \end{cases}$$

Then sketch by hand two representative trajectories (with their flow arrows).

Problem 6

Convert the given initial value problem into an initial value problem for the system

$$y'' = \cos(t)y(t) + y^2(t), \quad y(0) = 1 \quad y'(0) = 0.$$

Problem 7

Convert the given initial value problem into an initial value problem for the system

$$y^{(4)}(t) - y^{(3)}(t) + 7y(t) = \cos(t), \quad y(0) = 1, \quad y'(0) = 1, \quad y''(0) = 0, \quad y^{(3)}(0) = 2.$$

Problem 8

Solve the system

$$\begin{cases} \frac{dx}{dt} = 3x(t) - 4y(t) + 1 \\ \frac{dy}{dt} = 4x(t) - 7y(t) + 10t. \end{cases}$$

Problem 9

Write a scilab code to solve the second order differential equation

$$y'' = \cos(t)y(t) + y^2(t), \quad y(0) = 1 \quad y'(0) = 0$$

numerically. Your code should have three files: main program file, numerical method file and differential equation file.