Problem 1

Solve the given initial value problem

\[
\begin{align*}
\frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 9y &= t^2 e^{3t} \\
y(0) &= 2, \quad y'(0) = 17
\end{align*}
\]
Problem 2

Find a general solution to

\[
\frac{d^2y}{dx^2} - \frac{dy}{dx} = 4.
\]
Problem 3

Find a general solution to

\[
\frac{d^2y}{dt^2} + y = \cos(t)
\]
Problem 4

One solution to

$$2t^2 \frac{d^2y}{dt^2} + 3t \frac{dy}{dt} - y = 0, \ t > 0,$$

is $y_1(t) = \frac{1}{t}$. Use reduction of order to find $y_2(t)$. Then find a general solution.
Problem 5

Solve the system

\[
\begin{align*}
\frac{dx}{dt} &= y \\
\frac{dy}{dt} &= -9x + 6y.
\end{align*}
\]
Problem 6

Find the critical points of the system

\[
\begin{align*}
\frac{dx}{dt} &= y^2 - 3y + 2 \\
\frac{dy}{dt} &= (x - 1)(y - 2)
\end{align*}
\]

and solve the xy-phase plane equation.
Problem 7

Solve the following differential equation

\[ (y \cos(x) + 2xe^y)dx + (x^2e^y + \sin(x) - 1)dy = 0. \]
Problem 8

Find a general solution to the differential equation

\[ y'' + y = \tan(t). \]
Problem 9

Convert the given initial value problem into an initial value problem for the system

\[ y''' = \sin(t)y'(t) + y^2(t), \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1. \]
Problem 10

Consider the autonomous differential equation \( \frac{dy}{dt} = f(y) \) where \( f(y) = y(y/7 - 1)(1 - y/15) \).

(a) Sketch the graph of the solution to the initial value problem \( \frac{dy}{dt} = f(y), y(0) = 3 \). Find the \( \lim_{t \to +\infty} y(t) \).

(b) Sketch the graph of the solution to the initial value problem \( \frac{dy}{dt} = f(y), y(0) = -3 \). Find the \( \lim_{t \to +\infty} y(t) \).

(c) Sketch the graph of the solution to the initial value problem \( \frac{dy}{dt} = f(y), y(0) = 20 \). Find the \( \lim_{t \to +\infty} y(t) \).

(d) Sketch the graph of the solution to the initial value problem \( \frac{dy}{dt} = f(y), y(0) = 10 \). Find the \( \lim_{t \to +\infty} y(t) \).
Problem 11

Solve the system

\[
\begin{align*}
\frac{dx}{dt} &= -x + 3y \\
\frac{dy}{dt} &= 3x - y.
\end{align*}
\]
Problem 12

Find the solution to:

\[ y' + y = x, \quad y(0) = 4 \]
Problem 13

Find the solution to:

\[ y' = -\frac{x}{y}, \quad y(4) = -3 \]
Problem 14

Verify that $y_1(x) = x$ and $y_2(x) = 1/x$ are solutions of

$$x^2 y' + xy' - y = 0,$$

and then find the general solution of

$$x^2 y' + xy' - y = x \ln(x),$$

for $x > 0$
Problem 15

Solve the system

\[
\begin{align*}
\frac{dx}{dt} &= 2x - y \\
\frac{dy}{dt} &= x + 2y.
\end{align*}
\]