

Math 3120-001: Practice Final Examination

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Your Name:.....

Problem 1

Solve the given initial value problem using the method of laplace transform

$$\begin{cases} \frac{d^2y}{dt^2} + 6y = t^2 - 1 \\ y(0) = 0, y'(0) = -1 \end{cases}$$

Problem 2

Solve the given initial value problem using the method of laplace transform

$$\begin{cases} \frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 5y = te^t \\ y(0) = 2, y'(0) = -1 \end{cases}$$

Problem 3

Find a general solution to

$$\frac{d^2y}{dt^2} - y = xe^x.$$

Problem 4

Find a general solution to

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 5\frac{dy}{dt} = 0$$

Problem 5

One solution to

$$t^3 \frac{d^2 y}{dt^2} + t \frac{dy}{dt} - y = 0$$

is $y_1(t) = t$. Use reduction of order to find $y_2(t)$. Then find a general solution.

Problem 6

Find the critical points of the system

$$\begin{cases} \frac{dx}{dt} = x^2 - 1 \\ \frac{dy}{dt} = xy \end{cases}$$

and solve the xy-phase plane equation.

Problem 7

(a) Find a general solution

$$\begin{cases} \frac{dx}{dt} = -4x + y \\ \frac{dy}{dt} = -2x - y \end{cases}$$

(b) Sketch the phase portrait in the xy - plane.

(c) Is $(0,0)$ stable critical point? Explain your answer.

Problem 8

Solve the phase plane for the following system. Then sketch by hand two representative trajectories (with their flow arrows)

$$\begin{cases} \frac{dx}{dt} = (y - x)(y - 1) \\ \frac{dy}{dt} = (x - y)(x - 1) \end{cases}$$

Problem 9

Compute Laplace transform of $y(t)$ given that

$$\begin{cases} \frac{d^2y}{dt^2} + y = 1 + e^{2t} \\ y(0) = 1, y'(0) = 2 \end{cases}$$

Note that you are not asked to find $y(t)$.

Problem 10

Solve the following differential equation

$$ye^x dx + (y + e^x)dy = 0.$$

Problem 11

Find a general solution to the differential equation

$$y'' - 2y' + y = t^{-1}e^t.$$

Problem 12

Convert the given initial value problem into an initial value problem for the system

$$y'' = \cos(t)y(t) + y^2(t), \quad y(0) = 1 \quad y'(0) = 0.$$

Problem 13

Consider the autonomous differential equation $\frac{dy}{dt} = f(y)$ where $f(y) = y(y - 1)(y + 1)$.

- (a) Sketch the graph of the solution to the initial value problem $\frac{dy}{dx} = f(y), y(0) = 1/2$. Find the $\lim_{t \rightarrow +\infty} y(t)$.
- (b) Sketch the graph of the solution to the initial value problem $\frac{dy}{dt} = f(y), y(0) = 1/2$. Find the $\lim_{t \rightarrow +\infty} y(t)$.

Problem 14

Solve the system

$$\begin{cases} \frac{dx}{dt} = 2x(t) + y - e^{2t} \\ \frac{dy}{dt} = x + 2y. \end{cases}$$

Problem 15

Write a scilab code to solve the second order differential equation

$$y'' - x^2 y'(x) + y^2(x) = \sin(x), \quad y(2) = 1 \quad y'(2) = 0$$

numerically. Your code should have three files: main program file, numerical method file and differential equation file.

Problem 16

Find the solution to:

$$\cos^2(t) \sin(t)y' = -\cos^3(t)y(t) + 1, \quad y\left(\frac{\pi}{4}\right) = 0$$

Problem 17

Find the solution to:

$$y' = 1 + t^2 + y^2 + t^2 y^2 \quad y(0) = 1$$