

Math 1910-003: Practice Final Examination

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Your Name:.....

Problem 1

Find

$$\int 6x^2 (x^3 + 1)^4 dx$$

Problem 2

Evaluate

$$\int_{-1}^1 x (x^2 + 1)^3 dx$$

Problem 3

$$\int \frac{x + 10x^6}{x^4} dx$$

Problem 4

$$\int \frac{5x}{\sqrt{1+4x^2}} dx.$$

Problem 5

Use Newton's method to approximate a real zero of the function accurate to three decimal places.

$$f(x) = x^4 - 6x^2 + x + 5$$

using initial guess $x_1 = -4$

Problem 6

$$\int x^2 \sec^2(x^3) dx.$$

Problem 7

$$\int \sqrt{1 + \tan(x)} \sec^2(x) dx.$$

Problem 8

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2(x) \sec^2(x) dx.$$

Problem 9

Use the fact that

$$\int_3^5 4f(x) + 3dx = 26$$

to compute

$$\int_3^5 f(x)dx.$$

Problem 10

Use the Intermediate value theorem to show the equation

$$x^3 + 2x + 1 = 0 \text{ has a solution.}$$

Problem 11

Assume that $-x^4 \leq f(x) \leq x^2$. Find the limit

$$\lim_{x \rightarrow 0} f(x)?$$

Problem 12

Let $f(x) = x^2 - x + 1$. Compute the $f'(x)$ using

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

Problem 13

Find the derivative of

$$y = x^{\sin(x)},$$

then find $\frac{dy}{dx}$.

Problem 14

$$\int \frac{4x^4}{(1+x^5)^9} dx.$$

Problem 15

Use the Second Fundamental Theorem of Calculus to find $F'(x)$,

$$F(x) = \int_x^{x^2} t^2 \sqrt{1+t^3} dt$$

Problem 16

$$\int (x^2 + 1)(x - 3)dx.$$

Problem 17

$$\int \frac{1}{x} + \frac{1}{x^2} dx.$$

Problem 18

Sketch the graph of

$$f(x) = \frac{x}{x^2 - 1}.$$

Problem 19

Find the limit(if it exists)

(a)

$$\lim_{x \rightarrow 0} \frac{\sin(7x)}{4x}$$

(b)

$$\lim_{x \rightarrow 2} \frac{2 - x}{1 - \sqrt{x - 1}}$$

Problem 20

$$\int \tan(5x)dx.$$

Problem 21

Use the first derivative test to find any relative extreme of the function $f(x) = \frac{1}{3}x^3 - 4x$.

Problem 22

Find the derivative of the function $h(x) = \sqrt{e^{2x} + 4^{-2x}}$

Problem 23

Locate the absolute extrema of the function $f(x) = x^3 - 27x$ on the closed interval $[-5, 4]$

Problem 24

$$\int (2x)(x^2 - 2)dx.$$

Problem 25

Sketch the the region bounded by the graph of

$$y = f(x) = \frac{1}{x^2 - 1}, \quad y = 0, \quad x = 1, \quad x = 9,$$

and determine its area.

Problem 26

(a) Find $\lim_{x \rightarrow 1^-} f(x)$ where

$$f(x) = \begin{cases} \frac{x+2}{2} & x \leq 1 \\ \frac{12-2x}{3} & x > 1. \end{cases}$$

(b) Explain mathematically why the function

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & x \neq 1 \\ 4 & x = 1. \end{cases}$$

is not continuous at $x = 1$

Problem 27

$$\int \frac{5x^2}{1+x^2} dx.$$

Problem 28

- (a) Find the slope of tangent line to the graph of the function $f(x) = \sin(x + \frac{\pi}{2})$ at point $(0, 1)$
- (b) Find an equation of the tangent line to the graph of the function $f(x) = \sin(x + \frac{\pi}{2})$ at point $(0, 1)$

Problem 29

Find the derivative of $f(x)$

(a) Find the derivative of $f(x)$,

$$f(x) = \cos^2(x^2 - 5)$$

(b) if $xy + \cos(y) = x^2$, then $\frac{dy}{dx}$.