Risk Management and Hazard Model

for Tornadoes in

Rutherford County

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Definition:

According to the meteorological definition, a tornado is a violent rotating column of air extending from a cumulonimbus thunderstorm cloud to the ground. Every year more than 1,000 tornadoes hit the United States causing tremendous human and economic losses.

As threatening weather approaches, the storm may assume the form of brief down bursts, or the raging clouds may spin at hundreds of miles per hour. Tornadoes are destructive, often sudden, and always dangerous. People who have witnessed them know first-hand the eerie calm before the storm; the ascending assault of lightning and hail; and the winds that rip toward the earth with the sound of a freight train and the power of nothing else known in nature.

Introduction:

One of the principle applications of the tornado data is a tornado hazard assessment. Even though a tornado is a rare, localized event, modern society has become so complex that even the associated low risks have become important. With the knowledge of tornado *occurrence* and *intensity,* one can be prepared for a significant event. Areas with large populations will be able to develop better plans for tornado fury.

As the population increases, the damage due to a severe event such as a tornado, also increases. Through the use of a hazard model, tornado occurrence, location, path, length, path width, and F-scale (for intensity) can be modeled.

High concentrations of people in areas of increased tornado hazard require special planning. Officials at hospitals, schools, and factories should all be aware of their relative risks. Disaster plans should be prepared for the event of an emergency. Various minor modifications to the buildings can be performed to mitigate tornado effects. However, to make any such structural retrofitting economically feasible, it is necessary to estimate the damage probabilities.

Knowing how the tornado events vary year -to- year is extremely helpful in determining the overall risk. With the advent of nuclear reactors and associated facilities, the need for hazard assessment has moved from the realm of the insurance industry to that of the engineer, architect, regulator, and emergency planner. Using the variability and trends from the model, the risk of a significant tornado can be seen from any given location. With the output from this model, future changes in the climatology of tornado occurrence might be foreseen.

((Taken from Atmospheric Research, Volime 56, Nos. 1-4 Jan. 2001)) Tornadoes have been observed on all continents except Antarctica. In the 1960s, studies of the damage associated with individual tornadoes in the US led to the development of the Fujita damage scale (Fujita, 1971), a method to classify tornadoes based on the maximum level of damage. Although Fujita estimated windspeeds associated with the different levels of damage, in practice, the scale only provides information on damage.

All tornadoes in the US have been assigned F-scale values since 1973. It is possible to estimate F-scale value for tornadoes prior to that date if sufficient information is available. This has been done by Tecson et al. (1977) for all Us tornadoes back to 1916 and Grazulis (1993) for US strong and violent tornadoes as far back as 1640, and by other researchers for a number of other countries.

Several fundamental questions can be addressed by looking at the results from different countries. Chief among them is: What similarities and differences can be found in the distribution of tornadoes by damage around the world? If tornadoes have similar characteristics in different parts of the world, then it may be possible to use data from areas of relatively high frequency and quality of reports to make estimates of threats in other parts of the world.

The longest record collected by an official national agency at the time of the events occurs is that from the US, beginning in 1953 with the creation of the National Severe Storms Project.

Distributions by F-Scale

A feature of interest in the US record is that the distribution of tornadoes by F-scale has been approaching log-linear. This distribution is consistent with standard statistical distributions of rare events, such as the Gumble distribution, that show a nearly log-linear decline as the intensity of the event increases and the frequency at which it is observed decreases. This log-linear behavior has been seen in other weather records, such as extreme hourly precipitation amounts (Brook and Stensrud. 2000).

It is important to consider sources of error in the distribution. In general, there are at least four sources of error in the collection of data and classification of tornadoes by damage scale. First of all, there are times when no or very few reports at all are collected. (There is evidence of this in periods such as the 1940s in France and Germany, as well as mid-19th century in Germany). Second, low F-scale tornadoes are likely to be missed in the reporting because they typically have short lifetime and path length. Third, given that the assignment of an F-scale rating depends upon adequate structures being present to be damaged, it is likely that the number of tornadoes at the highest F-scale is underestimated. For example, if there are no structures present in the path of a tornado, it is impossible, in practice, to rate it as a violent tornado. In general, this kind of problem moves tornadoes from higher F-scale values towards lower F-scale values. Finally, there may be random errors in the assignment of F-scale.

Since the1950, the slope for US tornadoes has been relatively constant for F2 - F4 tornadoes. In the limiting case that the "true" distribution is characterized by a log-linear distribution, it can be shown that, for large numbers of reports, the slope of the line on a log-scale will not be affected by random classification errors except at the ends of the F-scale Since the other three kinds of errors do not affect the probability distribution function, the slope of the distribution between F2 and F4 is a basic parameter of the distributions. Between 27 and 35 F3 tornadoes have been reported annually in the US, on average, for every 100 F2 tornadoes, depending on decade, and between 5 and 8 F4 tornadoes have been reported for each 100 F2 tornadoes.

Additional insight into the nature of the distribution can be gained by looking at the number of tornadoes in different parts of the US for the period 1950-1995. In the Central Plains (the states of Oklahoma, Kansas, and Nebraska), a region roughly corresponding to an area sometimes called "tornado Alley", the number of F3 tornadoes per 100 F2 tornadoes is slightly more than 38, with 13 F4 tornadoes per 100 F2s. For the remainder of the US east of the region, except for Florida, the corresponding numbers are 34 and 13. There are almost 7000 tornadoes in the Central Plains region and over 17000 in the Eastern US region over the time period. The Eastern Us region is almost 10 times as large as the Central Plains, so that per unit area, there are about four times as many tornadoes in the Central Plains.}}

Tornado Database:

To perform such a hazard analysis, it is first necessary to obtain a complete, consistent, and statistical database detailing all available information about each individual occurrence. There is a 75 year data set (1921-1995), by Grazulis, that gives a good sampling of tornadoes, with approximately 10,000 tornadoes with damage intensity, path length and width data to build out hazard.

The National Severe Storms Forecast Center (NSSFC) and the Nuclear Regulatory Commission pursued newspaper accounts of all tornadoes reported over the United States. The NSSFC database contains information on 22,840 tornadoes, which occurred between 1950 and 1983 over the contiguous United States. Data was compiled on each tornado's path length, average path width, tornadic intensity, and monetary amount of damage produced by the storm. In the NSSFC database, 8.1% of the tornadoes are missing an intensity estimate. Since the intensity, length and width observations are required for risk analysis, 39.4% of the tornado reports are lacking enough information to preclude use. Adding ancillary data compiled by the Canadian Climatic Center increased the geographic coverage of the database. This data includes information on over 900 tornadoes that occurred across Canada during 1950 – 1979 inclusive.

In the development of the probability distributions for the tornado parameters, there is a database available by the Storm Prediction Center (SPC). This database, in particular, contains year, month, day, time, Fujita scale, length and width of the touchdown, as well as latitude and longitude of the starting and ending points of the tornado track. It must be noted that the determination of all of the primary parameters (intensity, path, width, and length) is to some extent subjective. The F-scale for tornado intensity is based on the worst – damage assessment.

For the United States, The mean length of a tornado is 7.1 kilometers with a standard deviation of 15.1 kilometers. The mean width is 117 meters with a standard deviation of 193 meters. If we are looking for the area we have the following: a mean of 1.68 kilometers squared and standard deviation of 6.96 kilometers squared (a very large standard deviation compared with the mean initiates a highly skewed distribution).

Since many more small tornadoes occur compared with the large ones, the median of tornadoes is a number more representative than that of the mean. This is typical of the United States tornado, which is 1.6 kilometers long, 43 meters wide and deviates 0.1 kilometers squared.

There are 38 extremely violent F5 tornadoes, which make up less than 0.2% of the total population. This class has the mean length of 55 kilometers (median length 37.7 km), mean width is 563 meters (median width of 454 meters), and mean area of 30.76 kilometers squared (median area 24.15 kilometers squared).

This distribution fit the data well, with the fits being best at F_2. By increasing the F-Scale we are facing the wider tornado. For F_2 – scale we observed the median width of about 100 meters, while for F_5 – scale tornado the median width is reached at 600 meters. If we are looking on the median length of tornado we have about 10 kilometers median length for F_2, and 60 kilometers for the F_5.

Surveys of various hazard models show a correlation between the length and width of the damaged area is typically postulated. Many of these models also hypothesize an explicit relationship between the damaged area and intensity. Since these correlations are the foundation of the model, their statistical relevance should be examined.

Tornado Hazard Model

There are five characteristics for each tornado, by knowing the statistical distributions of these characteristics, thousands of numbers can be described.

- First, the probability of a tornado occurring. This will depend upon the location that is being looked at; there are higher probabilities in areas where relatively high numbers of tornadoes occur.
- Second, the number of tornadoes that occur on a specific day, given that a tornado will occur. This can be taken from empirical fit to the observed number of tornadoes on days that tornadoes occurred from 1921 – 1995.
- Third, the F – Scale must determine based on the probability distribution function of each rating in their F – Scale statistics.
- The fourth and fifth elements are the length and width of the tornado's path, which are both taken from the Weibull distribution fits.

There is little literature devoted to the modeling of a tornado. In order to evaluate tornado risk, we need to assess tornado hazard, the damage due to the hazard, and the loss resulting from the damage. Thus, the three steps in risk analysis include hazard evaluation, damage estimation, and the loss assessment. The probability that a tornado will occur on any one day is fit by a Beta Distribution (McCormick 2000) and is fitted to the observed distribution of probabilities from Concannon et al. (2000).
If X is the Beta random variable with parameters α and β, then it has the following probability density function:

$$g(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1} \qquad \text{for} \quad 0 < X < 1$$
$$= 0 \qquad \text{otherwise}$$

The mean and variance for this distribution is

$$m = \frac{a}{a+b}$$

and

$$s^2 = \frac{ab}{(a+b+1)(a+b)^2}$$

respectively.

The Weibull distribution is found to be the best fit to the path length and path width. α and β, are two parameters are used to describe the curve. α describes the shape of the distribution, or where it peaks on the X-axis (for α = 1 the Weilbull distribution changes the exponential distribution with the only one parameter β). If α < 1, the curve resembles a backwards "J" shape and becomes more positively skewed. For

α > 1, the curve moves away from the Y-axis and becomes more sharply peaked. β parameter describes the *scale*, or "stretch" of the curve. For a given α, β works to either stretch or compress the curve along the X-axis. The formula for the Weibull CDF is:

$$F(x) = 1 - \exp[-(X/b)a].$$

The Weibull distribution has mean $\mu = \beta\Gamma(1+1/\alpha)$

and variance is $\sigma^2 = \beta^2 [\Gamma(1+2/\alpha) - \Gamma^2 (1+1/\alpha)]$.

For α=1, the mean is equal to β, and the variance simplifies to β^2.

By having the model of a tornado occurring, the risks can then be evaluated for specific areas of interest. With a definite hazard model, risk management officials, such as insurance companies and emergency response people, will have the knowledge of when tornadoes are most likely to occur in their areas, and they will be aware of the risk of a significant event on a specific day. Also, the public will be able to prepare for these events appropriately.

Mathamod

Mathamod is purely an empirical, minimum assumption for a tornado hazard assessment model. In its essence, the probability of a tornado striking a point is the ratio of the annual mean area covered by tornado to the area over which tornadoes may occur. The hazard probability is obtained by simply summing the damaged area of each tornado of intensity greater than the desired threshold, which was reported within a localized region. Then, this area is divided by the size of the region and the duration of the database.

The annual probability, P, is given:

$$P = \frac{\sum_{i=1}^{n} l_i w_i}{AY}$$

- **A** is the regional area
- **Y** is the number of years data available
- **L1** is the length of tornado "I"
- **W1** is its width and **n** is the number of tornadoes in area **A.**

Note: Since the intensity categorization (F-Scale) is entirely subjective, this rating system categorizes intensity estimates according to the amount, type, and appearance of tornado damage, a strong possibility of biases, inconsistencies and/or inaccuracies in the data exists.

Protecting Lives and Response Plan (EMERGENCY MANAGEMENT)

The local emergency manger must fight an annual battle to protect their communities from nature's most forceful phenomenon. This may be partially done by attributing to the region's emergency management efforts, which should combine response planning, human resources, technology, and public education to enable communities to protect lives that are otherwise helplessly placed in harm's way. For cities, it is simply a case of planning smartly and not being caught unprepared. "When a disaster does occur, the cost of recovery and mitigation of that disaster can be devastating if the planning before the event has not been proactive and effective".

The response plan covering the operations hierarchy includes law enforcement, fire and rescue, maintenance, communication, health and medical resources, and damage assessment. Everyone gets together with all of the emergency services and support people to figure out how they want to run the emergency response plan.

Preparing the Public

Although emergency mangers are charged with the task of doing everything they can to notify residents of impeding danger, residents have the responsibility of heeding the warning and taking it seriously.

To ensure preparedness for the safety during a tornado, emergency mangers have added public education to their list of duties. Such as, public education programs on where people should shelter. Managers attend schools, businesses, and governments to talk with them about safety. They give the recommendations as to what would most likely be the safest part of the building. They encourage residents to purchase flood and fire insurance. In addition to safety seminars, public education may encompass written materials and media coverage (i.e. – newspaper and television stations broadcasts of the educational event).

Even with the established plan, trained volunteers, technology, and public education the response to tornadoes is only as good as the person in charge. To know the plan and ensure its usefulness, the emergency manger must test it on a regular basis.

The very essence of the tornadoes, randomness and destruction, dictates the tornado response will never become second nature to emergency managers. However, tuning and practicing the plan can bring a sense of predictability to situations in which, chaos is sometimes the only remaining factor. As we pictured, a variety of resources can assist in tornado preparedness and reaction. If effectively coordinated, they can reduce losses and save lives when nature's fury is unleashed.

The Fujita Tornado Scale (F – Scale)

This scale relates the degree of damage to the intensity of the wind. Wind speed in a tornado ranges from values below that of hurricane speeds to more than 300 miles per hour. The maximal winds in tornadoes are often confined to extremely small areas and vary tremendously over very short distances, even within the funnel itself.

In 1971, Dr. T. Theodore Fujita from the University of Chicago, devised a six category scale to classify U.S. tornadoes into six intensity categories, named F_0-F_5. These categories are based upon the estimated maximum winds occurring within the funnel.

- F_0 – Gale or light tornado. It has light damage and the maximum wind speeds are 40-72 mph.
- F_1 – Moderate tornado, with moderate damage and has the maximum wind speeds between 73-112 mph.
- F_2 – Significant Tornado, with considerable damage with maximum wind speeds of 113-157 mph.
- F_3 – Severe Tornado, with severe damage. The maximum wind speeds for this tornado is between 158-206 mph.
- F_4 – Devastating Tornado, with devastating damage. This tornado has maximum wind speeds of 207-260.
- F_5 – Incredible Tornado. This tornado has incredible damage with maximum wind speeds of 261-318 mph.

Note:
Regional analysis of the distribution of tornadoes, by intensity, has shown that it is constant over the country. It is possible that there are real spatial differences.

Properties of some Distributions:

1- The Beta Distribution

Beta Function:

The two parameter integral defined by

$$B(a, b) = \int_0^1 y^{a-1}(1-y)^{b-1} dy, \quad \alpha > 0, \ \beta > 0$$

is called the beta function. This function is related to the gamma function in the following way.

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

This relation shows that B(α, β) is symmetrical with respect to the parameters α and β; that is

B(α, β) = B(β, α).

For various positive values of α and β, the beta function may be interpreted as the area under the function $f(x) = x^{a-1}(1-x)^{b-1}$ from x = 0 to x = 1. It is important to observe that the parameters α and β determine the shape of the curves.

When the random variable assumes values that are percentages or when concern is with physical phenomena of the continuos type, which have lying between 0 and 1, the beta distribution is used.

Definition: A random variable X is said to have a beta distribution if it has the following function for its probability density:

$$f(x; a, b) = \begin{cases} \dfrac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}, & 0 < x \leq 1, \ a, \ b > 0 \\ = \quad 0, & \text{elsewhere.} \end{cases}$$

The two parameters α and β determine the shape of the beta distribution. Figure 1 shows the graph of the density for a fixed value of β, while α varies.

f{(x,3, 3), f(x, 4, 3) , f(x, 8, 3)}, x=0..1);

0

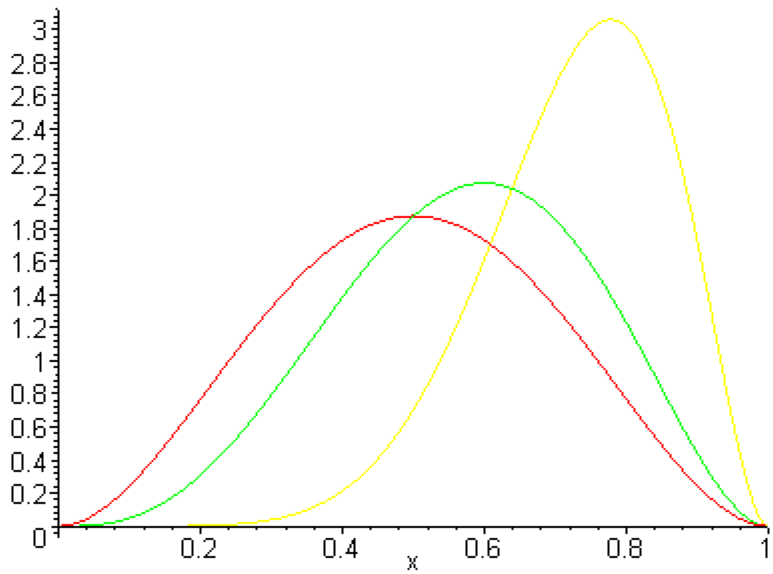
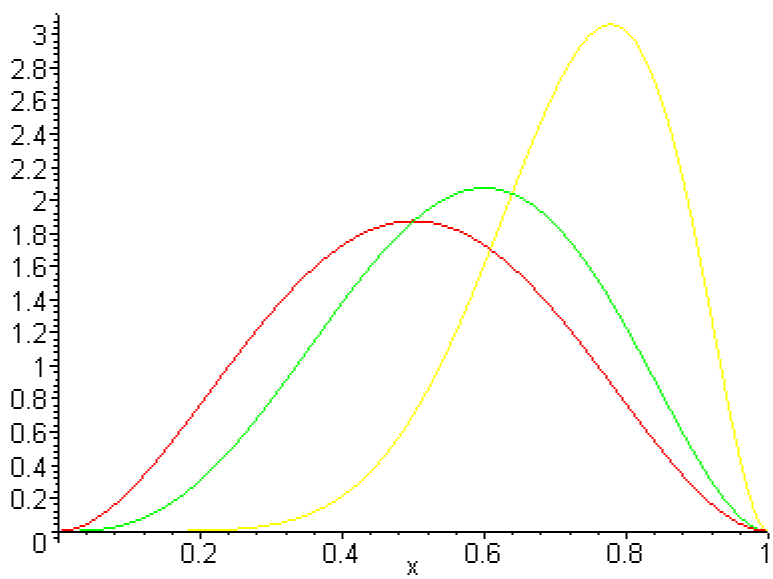


Figure 1

In Figure 2, we show the graph of the density for a fixed value of α, while β varies.

f{(x, 2, 4), f(x, 2, 3) , f(x, 2, 2)}, x=0..1);



Observe that, when $\alpha < \beta$ the density is skewed to the right and, when $\alpha > \beta$, it is skewed to the left. The beta distribution is symmetrical when $\alpha = \beta$. The graph of the distribution is U-shaped when $(\alpha - 1)$ and $(\beta - 1)$ are negative and J-shaped if only one of them is negative. The density function attains its mode at $x = (\alpha - 1)/(\alpha + \beta - 2)$ when $(\alpha - 1)$ and $(\beta - 1)$ are positive; the same point is its minimum when they are negative. It is possible to generate a family of beta density functions, all of which will have the same mode, by selecting the values of the parameters α and β properly.

Some special cases of the beta distribution are:

a) When $\alpha = \beta = 1$, we have the rectangular (uniform) distribution,

$$f(x) = 1, \quad 0 \le x \le 1,$$
$$= 0, \quad \text{elsewhere.}$$

b) When $\alpha = 1, \beta = 2$, and $\alpha = 2, \beta = 1$, we have the triangular distributions,

$$f(x; \alpha = 1, \beta = 2) = 2(1 - x), \quad 0 \le x \le 1,$$
$$= 0, \quad \text{elsewhere}$$

and

$$f(x; \alpha = 2, \beta = 1) = 2x, \quad 0 \le x \le 1,$$
$$= 0, \quad \text{elsewhere}$$

2- The Log-Normal Distribution

The log-normal distribution in its simplest form may be defined as the distribution of a random variable whose logarithm obeys the normal probability density function. Let Y be a random variable that is normally distributed, with parameters μ_y and σ_y. If $Y = \ln X$ or $X = e^Y$, then X is said to have a log-normal distribution. This distribution arises in physical problems when the domain of the variate, X, is greater than zero and its histogram is markedly skewed. This skewed occurs when X is affected by random causes that produce small effects that are proportional to the variate X. The outcome of these random causes, each producing a small constant effect, is normally distributed. The log-normal distribution has been used in economics, sociology, biology, and anthropometry, and in various physical and industrial processes.

Definition: A random variable X is distributed as a log-normal if it has the following function for its probability density:

$$f(x) = \frac{1}{x\sigma_y\sqrt{2\pi}} \exp\left\{ -\frac{1}{2\sigma_y^2}(\ln x - \mu_y)^2 \right\}, \qquad \rightarrow x > 0,\ \sigma_y > 0,\ -\infty < \mu_y < \infty,$$

$$= 0 \qquad \text{elsewhere.}$$

Figures 3 and 4 show the graph of the log-normal density for fixed μ_y and varying σ_y^2 , and for fixed σ_y^2 and varying μ_y, respectively.

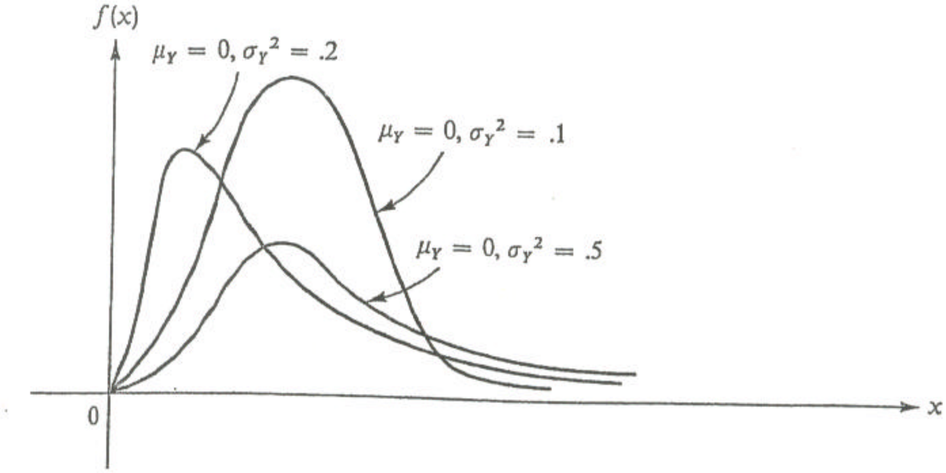


Figure 3

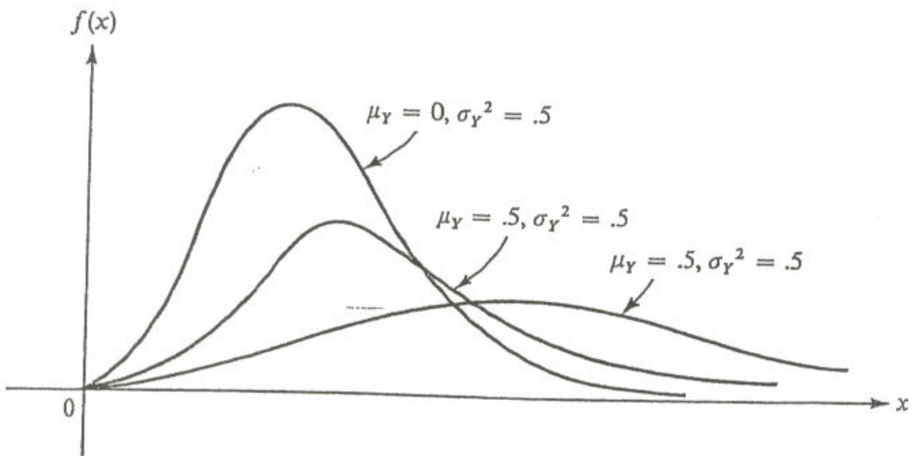


Figure 4

Figures 3 and 4 illustrate that the distribution is positively skewed and that, the greater the value of the parameter σ_y , the greater the skewness.

The median of the log-normal is at x = exp(μ_y). The relative position of the mean, median, and mode are at $e^{\mu_y + (1/2)\sigma_y^2}, e^{\mu_y}, and, e^{\mu_y - \sigma_y^2}$, respectively, where the mean is the average value of X,

$$\int_{-\infty}^{\infty} x f(x) dx,$$

the median is that value of m which satisfies the equation

$$\int_{-\infty}^{m} f(x) dx = 1/2 = \int_{m}^{\infty} f(x) dx,$$

and the mode is the value of x at which the probability density function attains its maximum.

The log-normal distribution possesses a number of interesting properties, most of which are immediate consequences of the normal distribution. The question of when the log-normal distribution is applicable in a given physical problem when a certain amount of data has been obtained can be answered by plotting the cumulative distribution of lnX on normal probability paper; if the resulting curve is nearly a straight line, then X has a log-normal distribution. In such a problem, the parameters μ_y and σ_y are estimated from the given information.

3- The Weibull Distribution

This distribution is applicable to many physical phenomena. It is extremely useful in studies of failure models.

Definition: A random variable X is said to be distributed as the Weibull distribution if it has the following function for its probability density:

$$f(x; \alpha, \beta, \gamma) = \frac{\beta}{\alpha}(x-\gamma)^{\beta-1} e^{-\left\{ \frac{(x-\gamma)^{\beta}}{\alpha} \right\}} \qquad x > \gamma, \quad \alpha, \ \beta, \ \gamma \ > 0,$$

$$= 0, \qquad \text{elsewhere.}$$

The three parameters α, β, and γ, which completely describe the Weibull density, are of significant importance. Here α is a scale parameter, β is a shape parameter, and γ is the location or threshold parameter. The density function attains its maximum point when

$$x = \gamma + \frac{\gamma(\beta - 1)^{1/\beta}}{\beta}$$

Figure 5 shows a graphical representation of the Weibull density for $\alpha = 1$, $\gamma = 0$, and varying β, $\beta = ½, 1, 2, 5$.

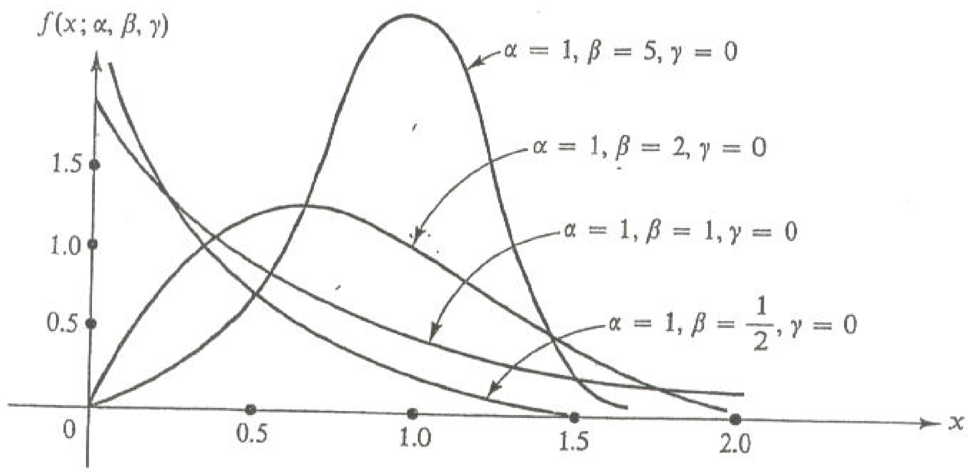


Figure 5

We notice that the density is skewed to the right for all values of the parameters. However, the skewness decreases as the shape parameter β increases. The cumulative distribution of the Weibull for $x > \gamma$ is given by

$$F(x) = 1 - e^{\{(x-g)^{b/a}\}}, \quad x > \gamma, \quad \alpha, \beta > 0, \quad \gamma \geq 0$$
$$= 0 \quad , \quad x \leq 0.$$

The Weibull density function is flexible enough to be applicable to a number of problems. This flexibility can be displayed by deriving the following distributions as special cases of the Weibull:

a) ***When b = 1 and g = 0 in the Weibull density, we obtain the exponential distribution,***

$$f(x, a) = 1/a \; e^{-(x/a)}, x \geq 0, \; a \rangle 0,$$
$$= 0 \quad , \text{elsewhere.}$$

b) ***When b = 2 and g = 0, we obtain Rayleigh distribution, with the following density function***:

$$f(x) = \frac{2}{a} x e^{-(x^2/a)}, \quad x \geq 0, \; a \rangle 0,$$
$$= 0, \quad \text{elsewhere.}$$

This distribution is significant importance in the theory of sound. Its meaning and derivation can be obtained by considering the following problem: In trying to locate an object on the x y plane, we determine its distance from the origin by measuring the distance along the x and y axes and applying the Pythagorean formula: $r^2 = x^2 + y^2$. If the measurements are subject to random error with X and Y representing errors in measurement that are assumed to be independent and normally distributed with $\mu = 0$ and $\sigma^2 = \alpha/2$, then the distribution of $R = \sqrt{X^2 + Y^2}$ is known as the Rayleigh distribution.

Figure 6 shows a graphical representation of the behavior of the shape parameter α.

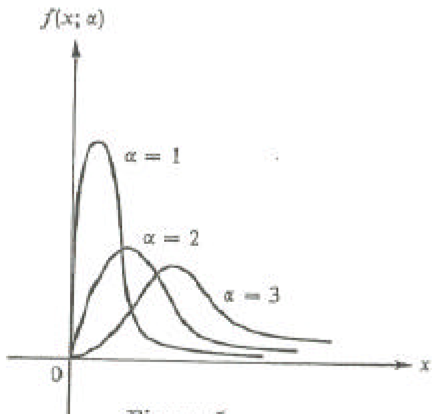


Figure 6

c) The Weibull distribution offers a very close approximation to the normal distribution for certain values of the parameters. It was found in a particular problem that, for $\alpha = 1$, . $\beta = 3.2589$, and $\gamma = 0$ of the Weibull density and for $\mu = 0.8964$ and $\sigma^2 = .0924$ of the Gaussian density, the two are practically identical. This comparison is shown by Figure 7.

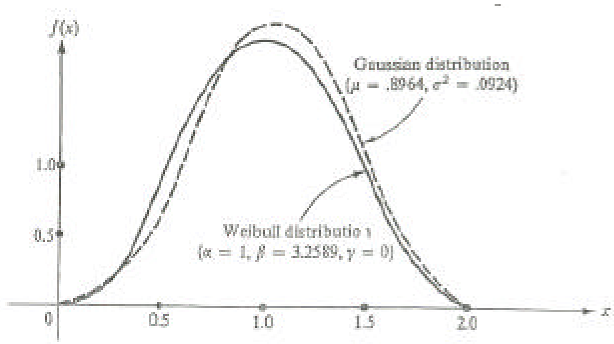


Figure 7