Chapter 3: Force and Motion

Homework: All questions on the “Multiple-Choice” and the odd-numbered questions on “Exercises” sections at the end of the chapter.

Force and Motion – Cause and Effect

- In chapter 2 we studied motion but not its cause.
- In this chapter we will look at both force and motion – the cause and effect.
- We will consider Newton’s:
  - Three laws of motion
  - Law of universal gravitation
  - Laws of conservation of linear and angular momentum

Sir Isaac Newton (1642 – 1727)

- Only 25 when he formulated most of his discoveries in math and physics
- His book *Mathematical Principles of Natural Philosophy* is considered to be the most important publication in the history of Physics.

Force and Net Force

- Force – a quantity that is capable of producing motion or a change in motion
  - A force is capable of changing an object’s velocity and thereby producing acceleration.
- A given force may not actually produce a change in motion because other forces may serve to balance or cancel the effect.

Balanced (equal) forces, therefore no motion.

Equal in magnitude but in opposite directions.

Unbalanced forces result in motion

Net force to the right
Newton’s First Law of Motion

• Aristotle considered the natural state of most matter to be at rest.
• Galileo concluded that objects could naturally remain in motion.
• Newton – An object will remain at rest or in uniform motion in a straight line unless acted on by an external, unbalance force.

Objects at Rest

• An object will remain at rest or in uniform motion in a straight line unless acted on by an external, unbalance force.
• Force – any quantity capable of producing motion
• Forces are vector quantities – they have both magnitude and direction.
• Balanced → equal magnitude but opposite directions
• External → must be applied to the entire object or system.

A spacecraft keeps going because no forces act to stop it

Inertia

• Inertia - the natural tendency of an object to remain in a state of rest or in uniform motion in a straight line (first introduced by Galileo)
• Basically, objects tend to maintain their state of motion and resist changes.
• Newton went one step further and related an object’s mass to its inertia.
  – The greater the mass of an object, the greater its inertia.
  – The smaller the mass of an object, the less its inertia.

A large rock stays put until/if a large enough force acts on it.

Mass and Inertia

The large man has more inertia – more force is necessary to start him swinging and also to stop him – due to his greater inertia.
Mass and Inertia

Quickly pull the paper and the stack of quarters tend to stay in place due to inertia.

“Law of Inertia”

- Because of the relationship between motion and inertia:
- Newton’s First Law of Motion is sometimes called the Law of Inertia.
- Seatbelts help ‘correct’ for this law during sudden changes in speed.

Newton’s Second law of Motion

• *Acceleration* $\alpha \frac{\text{Force}}{\text{mass}}$

  - Acceleration (change in velocity) produced by a force acting on an object is directly proportional to the magnitude of the force (the greater the force the greater the acceleration.)
  - Acceleration of an object is inversely proportional to the mass of the object (the greater the mass of an object the smaller the acceleration.)
  - $a = \frac{F}{m}$ or $F = ma$

Force, Mass, Acceleration

- a) Original situation $a \propto \frac{F}{m}$
- b) If we double the force we double the acceleration.
- c) If we double the mass we half the acceleration.

Net Force and Total Mass - Example

- Forces are applied to blocks connected by a string (weightless) resting on a frictionless surface. Mass of each block = 1 kg; $F_1 = 5.0$ N; $F_2 = 8.0$ N
- What is the acceleration of the system?

$F = ma$

- “F” is the net force (unbalanced), which is likely the vector sum of two or more forces.
- “m” & “a” are concerning the whole system
- Units
  - Force = mass x acceleration = kg x m/s² = N
  - N = kg-m/s² = newton -- this is a derived unit and is the metric system (SI) unit of force

Units

$F_1 = -5.0$ N

$F_2 = 8.0$ N

$a = \frac{F}{m}$
Net Force and Total Mass - Example

- Forces are applied to blocks connected by a string (weightless) resting on a frictionless surface. Mass of each block = 1 kg; $F_1 = 5.0$ N; $F_2 = 8.0$ N. What is the acceleration of the system?
- GIVEN:
  - $m_1 = 1$ kg; $m_2 = 1$ kg
  - $F_1 = -5.0$ N; $F_2 = 8.0$ N
- $a = ?$
- $a = \frac{F}{m} = \frac{F_{\text{net}}}{m_1 + m_2} = \frac{8.0 \text{ N} - 5.0 \text{ N}}{1.0 \text{ kg} + 1.0 \text{ kg}} = 1.5 \text{ m/s}^2$

Mass & Weight

- Mass = amount of matter present
- Weight = related to the force of gravity
- Earth: weight = mass x acc. due to gravity
  - $w = mg$ (special case of $F = ma$) Weight is a force due to the pull of gravity.
- Therefore, one’s weight changes due to changing pull of gravity – like between the earth and moon.
  - Moon’s gravity is only $1/6$th that of earth’s.

Computing Weight – an example

What is the weight of a 2.45 kg mass on (a) earth, and (b) the moon?

- Use Equation $w = mg$
- Earth: $w = mg = (2.45 \text{ kg}) (9.8 \text{ m/s}^2) = 24.0 \text{ N}$ (or 5.4 lb. Since 1 lb = 4.45 N)
- Moon: $w = mg = (2.45 \text{ kg}) [(9.8 \text{ m/s}^2)/6] = 4.0 \text{ N}$ (or 0.9 lb.)

Newton’s Third Law of Motion

- For every action there is an equal and opposite reaction.
  - $F_1 = -F_2$ or $m_1a_1 = -m_2a_2$
Newton’s Third Law of Motion

- \( F_1 = -F_2 \) or \( m_1a_1 = -m_2a_2 \)
- Jet propulsion – exhaust gases in one direction and the rocket in the other direction
- Gravity – jump from a table and you will accelerate to earth. In reality BOTH you and the earth are accelerating towards each other
  - You – small mass, huge acceleration \( (m_1a_1) \)
  - Earth – huge mass, very small acceleration \( (-m_2a_2) \)
  - BUT \( m_1a_1 = -m_2a_2 \)

Newton’s Laws in Action

- Friction on the tires provides necessary centripetal acceleration.
- Passengers continue straight ahead in original direction and as car turns the door comes toward passenger – 1st Law
- As car turns you push against door and the door equally pushes against you – 3rd Law

Newton’s Law of Gravitation

- Gravity is a fundamental force of nature
  - We do not know what causes it
  - We can only describe it
- Law of Universal Gravitation – Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

- \( F = \frac{Gm_1m_2}{r^2} \)
- \( G \) is the universal gravitational constant
  - \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \)
- \( G \):
  - is a very small quantity
  - thought to be valid throughout the universe
  - was measured by Cavendish 70 years after Newton’s death
  - not equal to “g” and not a force

For a homogeneous sphere the gravitational force acts as if all the mass of the sphere were at its center

\[ F = \frac{Gm_1m_2}{r^2} \]
Applying Newton’s Law of Gravitation

- Two objects with masses of 1.0 kg and 2.0 kg are 1.0 m apart. What is the magnitude of the gravitational force between the masses?

\[
F = \frac{Gm_1m_2}{r^2}
\]

\[
F = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.0 \text{ kg})(2.0 \text{ kg})}{(1.0 \text{ m})^2}
\]

\[
F = 1.3 \times 10^{-10} \text{ N}
\]

Applying Newton’s Law of Gravitation – Example

- Two objects with masses of 1.0 kg and 2.0 kg are 1.0 m apart. What is the magnitude of the gravitational force between the masses?

\[
F = \frac{Gm_1m_2}{r^2}
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F = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.0 \text{ kg})(2.0 \text{ kg})}{(1.0 \text{ m})^2}
\]

\[
F = 1.3 \times 10^{-10} \text{ N}
\]

Force of Gravity on Earth

- \( F = \frac{GmM_E}{R_E^2} \) [force of gravity on object of mass \( m \)]
- \( M_E \) and \( R_E \) are the mass and radius of Earth
- This force is just the object’s weight \( (w = mg) \)

\[
\Rightarrow w = mg = \frac{GM_E}{R_E^2}
\]

\[
g = \frac{GM_E}{R_E^2}
\]

\( m \) cancels out \( \Rightarrow g \) is independent of mass

Acceleration due to Gravity for a Spherical Uniform Object

- \( g = \frac{GM}{r^2} \)
- \( g \) = acceleration due to gravity
- \( M \) = mass of any spherical uniform object
- \( r \) = distance from the object’s center

“Weightlessness” in space is the result of both the astronaut and the spacecraft ‘falling’ to Earth as the same rate
Linear Momentum

- Linear momentum = mass x velocity
  \[ r = mv \]
- If we have a system of masses, the linear momentum is the sum of all individual momentum vectors.
  \[ P = P_1 + P_2 + P_3 + \ldots \] (sum of the individual momentum vectors)

Law of Conservation of Linear Momentum

- Law of Conservation of Linear Momentum - the total linear momentum of an isolated system remains the same if there is no external, unbalanced force acting on the system.
- Linear Momentum is ‘conserved’ as long as there are no external unbalance forces.
  - It does not change with time.

Conservation of Linear Momentum

- \( P_i = P_f = 0 \) (for man and boat)
- When the man jumps out of the boat he has momentum in one direction and, therefore, so does the boat.
- Their momentums must cancel out! (= 0)

Applying the Conservation of Linear Momentum

- Two masses at rest on a frictionless surface. When the string (weightless) is burned the two masses fly apart due to the release of the compressed (internal) spring \((v_1 = 1.8 \text{ m/s})\).

GIVEN:

- \( m_1 = 1.0 \text{ kg} \)
- \( m_2 = 2.0 \text{ kg} \)
- \( v_1 = 1.8 \text{ m/s}, \quad v_2 = ? \)

- \( P_f = P_i = 0 \)

- \[ v_2 = \frac{-m_1 v_1}{m_2} \]

\( m_1 v_1 = -m_2 v_2 \)

\[ v_2 = \frac{-1.0 \text{ kg} \times 1.8 \text{ m/s}}{2.0 \text{ kg}} = -0.90 \text{ m/s} \]
Jet Propulsion

- Jet Propulsion can be explained in terms of both Newton’s 3\textsuperscript{rd} Law & Linear Momentum
  \[ \rho_1 = -\rho_2 \Rightarrow m_1 v_1 = -m_2 v_2 \]
- The exhaust gas molecules have small \( m \) and large \( v \).
- The rocket has large \( m \) and smaller \( v \).
- BUT \( m_1 v_1 = -m_2 v_2 \) (momentum is conserved)

Angular Momentum

- \( L = mvr \)
- \( L \) = angular momentum, \( m \) = mass, \( v \) = velocity, and \( r \) = distance to center of motion
- \( L_1 = L_2 \)
- \( m_1 v_1 r_1 = m_2 v_2 r_2 \)

Torque

- Torque – the twisting effect caused by one or more forces
- As we have learned, the linear momentum of a system can be changed by the introduction of an external unbalanced force.
- Similarly, angular momentum can be changed by an external unbalanced torque.

Law of Conservation of Angular Momentum

- Law of Conservation of Angular Momentum - the angular momentum of an object remains constant if there is no external, unbalanced torque (a force about an axis) acting on it
- Concerns objects that go in paths around a fixed point, for example a planet orbiting the sun

Angular Momentum

- Mass (\( m \)) is constant.
- As \( r \) changes so must \( v \). When \( r \) decreases, \( v \) must increase so that \( m_1 v_1 r_1 = m_2 v_2 r_2 \)
Angular Momentum in our Solar System

- In our solar system the planet’s orbit paths are slightly elliptical, therefore both \( r \) and \( v \) will slightly vary during a complete orbit.

Example of Conservation of Angular Momentum

- A comet at its farthest point from the Sun is 900 million miles, traveling at 6000 mi/h. What is its speed at its closest point of 30 million miles away?
- EQUATION: \( m_1v_1r_1 = m_2v_2r_2 \)
- GIVEN: \( v_2, r_2, r_1 \), and \( m_1 = m_2 \)
- FIND: \( v_1 = \frac{v_2r_2}{r_1} = \frac{(6.0 \times 10^3 \text{ mi/h}) (900 \times 10^6 \text{ mi})}{30 \times 10^6 \text{ mi}} = 1.8 \times 10^5 \text{ mi/h or 180,000 mi/h} \)

Conservation of Angular Momentum

- Rotors on large helicopters rotate in the opposite direction

Conservation of Angular Momentum

- Figure Skater – she/he starts the spin with arms out at one angular velocity. Simply by pulling the arms in the skater spins faster, since the average radial distance of the mass decreases.
  - \( m_1v_1r_1 = m_2v_2r_2 \)
  - \( m \) is constant; \( r \) decreases;
  - Therefore \( v \) increases

Chapter 3 - Important Equations

- \( F = ma \) (2nd Law) or \( w = mg \) (for weight)
- \( F_1 = -F_2 \) (3rd Law)
- \( F = \frac{(Gm_1m_2)}{r^2} \) (Law of Gravitation)
- \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \) (gravitational constant)
- \( g = GM/r^2 \) (acc. of gravity, \( M \)=mass of sph. object)
- \( r = mv \) (linear momentum)
- \( P_1 = P_2 \) (conservation of linear momentum)
- \( L = mv^2 \) (angular momentum)
- \( L_1 = m_1v_1r_1 = L_2 = m_2v_2r_2 \) (Cons. of ang. Mom.)