Contract Rate Versus Effective Cost
Chapter 4

You are asked to evaluate 3 mortgage loans for a friend who wishes to borrow $100,000 using a fixed-rate, monthly payment, 30 year loan. The following loan offerings are available:

<table>
<thead>
<tr>
<th>Loan</th>
<th>Nominal % Rate</th>
<th>Disc. pts. / Orig. pts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.63%</td>
<td>0/0</td>
</tr>
<tr>
<td>2</td>
<td>6.25%</td>
<td>1/1</td>
</tr>
<tr>
<td>3</td>
<td>6.00%</td>
<td>2/1</td>
</tr>
</tbody>
</table>

1. What are the effective costs of these loans?
2. If after five years your friend expected to be transferred and therefore sell the house and prepay the loan, what would the effective costs have actually been on these loans?

1.)

<table>
<thead>
<tr>
<th>Loan</th>
<th>Monthly Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$640.64</td>
</tr>
<tr>
<td>2</td>
<td>$615.72</td>
</tr>
<tr>
<td>3</td>
<td>$599.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Actual Disbursement ($)</th>
<th>Effective Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>6.63%</td>
</tr>
<tr>
<td>$98,000</td>
<td>6.44%</td>
</tr>
<tr>
<td>$97,000</td>
<td>6.29%</td>
</tr>
</tbody>
</table>

2.)

<table>
<thead>
<tr>
<th>Loan</th>
<th>OLB after 60th payment</th>
<th>Effective Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$93,749.52</td>
<td>6.63%</td>
</tr>
<tr>
<td>2</td>
<td>$93,337.67</td>
<td>6.74%</td>
</tr>
<tr>
<td>3</td>
<td>$93,054.28</td>
<td>6.73%</td>
</tr>
</tbody>
</table>
CHARGING FEES (POINTS) TO ACHIEVE YIELD
"Pricing" a Loan (text pp. 101-102)

A lender wants to offer an "attractive" nominal (contract) rate, a rate that is lower than the lender's required (effective) yield. How should the lender "price" their loan? If they offer a lower contract rate, what amount of discount and/or origination fees must be charged to provide the lender's required yield?

This problem can be solved using any assumed loan amount. Other required information includes- contract rate, required effective rate, amortization term, and an estimate of actual loan life.

Example: Assume a lender wishes to offer a 6 percent contract rate, but desires an effective yield of 6 1/2 percent. What would the loan disbursement be, and how many discount/origination points should be charged?

1. Assume a 30 year loan amortization and loan life.

\[ PV = (\$1); \ I = 6\%; \ n = 360; \ pmt = ? \]
\[ pmt = 0.005996 \]

\[ pmt = 0.005996; \ I = 6 \ 1/2 \%; \ n = 360; \ PV = ? \]
\[ PV = 0.948554 = \text{disbursement} \]

Therefore, the lender could disburse 0.948554 of every dollar loaned to achieve the required 6 1/2% return. If the loan is priced by offering terms of 6% and a 5.1446% orig./disc. fee (100% - 94.8554%), and the loan is not prepaid, the lender will have the required 6 1/2% yield.

2. What if the lender felt the loan would be prepaid after 10 years?

\[ PV = (\$1); \ I = 6\%; \ n = 360; \ pmt = ? \]
\[ pmt = 0.005996 \]

\[ I = 6\%; \ n = 240; \ pmt = 0.005996; \ PV = ? \]
\[ PV = 0.836926 = \text{OLB at end of 10th year} \]

\[ FV = 0.836926; \ I = 6 \ 1/2\%; \ n = 120; \ pmt = 0.005996; \ PV = ? \]
\[ PV = 0.96574 = \text{disbursement} \]

Therefore, 100% - 96.574% = 3.426% to be charged in fees/points to achieve a 6 1/2% effective rate of return.

Why are less fees/points charged in this scenario?
VARIATIONS ON MORTGAGE PROBLEMS- Chapter 4

ASSUMPTIONS: $100,000 LOAN; 12% INTEREST RATE; 20 YEAR MATURITY (MONTHLY PAYMENTS)

1. Borrower can't make required payment ($1,101.10), but agrees to pay $1,000 per month. The loan will be refinanced at the end of five years. If the lender requires a 12% return, what balloon payment will be necessary? What if the monthly payment were $950?

2. Borrower and lender agree on a balloon payment of $75,000 at the end of the fifth year. What should the monthly payments be to provide the lender with a twelve percent return?

3. What happens if part of the loan is prepaid? Assume the borrower prepays $10,000 at the end of the fifth year. How many months will be required to pay this loan off? How much interest would be saved versus no prepayment?

4. Assume the borrower wants to increase her monthly payment by $100 per month. What effect will this have on the number of payments needed to pay the loan off?

5. You calculate your monthly payment based on the assumptions above ($1101.10). You, however, plan to make 13 payments per year. How long will it take to pay the loan off?

6. You decide to pay one half of the calculated monthly mortgage payment every 2 weeks (26 payments per year). How long will it take to pay the loan off?
Solutions to: Variations on Mortgages Problems- Chapter 4

1. a. $100,000
   \[ PV = \frac{100,000}{(1+0.12)^{60}}; \quad n=60; \quad i=12\%; \quad \text{pmt.} = \$1,000; \quad FV = ? \]
   b. $104,083.48 (an example of negative amortization)
   \[ PV = \frac{100,000}{(1+0.12)^{60}}; \quad n=60; \quad i=12\%; \quad \text{pmt.} = \$950; \quad FV = ? \]

2. $1306.11
   \[ PV = \frac{100,000}{(1+0.12)^{60}}; \quad n=60; \quad i=12\%; \quad FV = \$75,000; \quad \text{pmt.} = ? \]

3. a. 136.31 more months; therefore, 196.31 total pmts versus 240 scheduled pmts
   \[ OL balance = \frac{91,745.48}{(1+0.12)^{60}} \]
   - $10,000 prepaid
   \[ PV = \frac{81,745.48}{(1+0.12)^{60}}; \quad i=12\%; \quad \text{pmt.} = \$1101.10; \quad n=? \]
   b. $38,107.06 interest savings
   \[ 240 \text{ payments } \times \$1101.10 = \$264,264 \]
   - $164,264 int.
   \[ 196.31 \text{ payments } \times \$1101.10 = \$216,156.94 \]
   - $10,000 prepayment
   - $100,000 prin.
   Therefore, $164,264 - $126,156.94 = $38,107.06 interest savings

4. 179.61 months
   \[ PV = \frac{100,000}{(1+0.12)^{n}}; \quad i=12\%; \quad \text{pmt.} = \$1201.10; \quad n=? \]

5. 198.31 Payments (assumes 13 regularly scheduled pmts./yr.)
   198.31/13 = 15.25 years
   \[ PV = \frac{100,000}{(1+0.12)^{n}}; \quad i=12\%; \quad \text{pmt.} = \$1101.10; \quad n=? \]
   - Change pmts. /yr. to 13
   - (Payment is larger than would be required to amortize loan over 20 years with 13 payments per year. Additionally, payments are being received sooner than old schedule.)

6. 393.7694 payments (393.7694/26= 15.145 years
   This is often called a bi-monthly mortgage.
   $1101.10/2 = $550.55 every two weeks; 52 weeks in a year; Therefore, 26 payments/year are made.
   \[ PV = \frac{100,000}{(1+0.12)^{n}}; \quad i=11.9679\%; \quad \text{pmt.} = $550.55; \quad n=? \quad *I=12\% \text{ cpd. mo.}= 12.6825\% \]
   annual effective rate = 11.9679\% cpd.26 times/year
   Interest Saved?
   \[ $1101.10 \times 240 = \$264,264 \quad \text{versus} \quad $550.55 \times 393.7694 = \$216,789.74 \]
   - $100,000 prin.
   - $100,000 prin.
   \[ $164,264 \text{ int.} \quad \text{versus} \quad $116,789.74 \text{ int.} \]
FIN 444- Real Estate Finance

Mortgage Problems

1. You can make payments of $1,000/month. If the lender requires a 10% return, how much would she lend you on a ten years loan?

2. A lender requires an 8% rate of return. They want to quote a nominal rate of 7 ½% on their mortgage. How many discount points must be charged to provide the required rate of return assuming the loan is amortized monthly over 15 years?

3. You borrow $100,000 at an 8% interest rate, and a 20 years term (monthly payments)? Assume you will be charged 1 origination point and 3 loan discount points. What is the effective rate of interest? If this loan were prepaid after 10 years what would be the effective cost? If the loan will be prepaid at the end of year 10, but will be subject to a 5% prepayment penalty, what will the effective yield be to the lender?

4. You borrowed $100,000 today to be repaid in monthly payments over the next 30 years. The interest rate is 10%. You expect to pay down the OLB by $20,000 at the end of the tenth year. If you refinance the OLB, less the $20,000, at that time at 8% for 20 years (monthly payments), what will your payments be? How much would you still owe after ten years of repayment on the second loan? How much total interest will you pay over the 30 years period? What amount of interest would be saved by paying down the loan and refinancing at the lower interest rate?
FIN 444 - Real Estate Finance

Mortgage Problem Solutions

1. \( Pmt=1000 \)
   \( l=10\% \)
   \( N=120 \)
   \( Pv=?=75,671.16 \)

2. \( Pv=(1) \)
   \( N=180 \)
   \( I=7.5\% \)
   \( Pmt=?=0.00927012 \)
   \( Pmt=0.00927012 \)
   \( N=180 \)
   \( I=8\% \)
   \( Pv=?=97003122 \)
   \( 1-.97003122=.02996818=approximately\space 3\space points \)

3. \( Pv=(100,000) \)
   \( I=8\% \)
   \( N=240 \)
   \( Pmt=836.44 \)
   \( \frac{Pmt}{836.44} \)
   \( N=240 \)
   \( Pmt=836.44 \)
   \( \frac{I}{8.55\%} \)
   \( Pmt=836.44 \)
   \( N=120 \)
   \( Pmt=836.44 \)
   \( Fv=OLB=68,941 \)
   \( I=8.68\% \)
   \( Pmt=836.44 \)
   \( N=120 \)
   \( Pmt=836.44 \)
   \( Fv=OLB+\text{penalty}=72,388 \)
   \( I=8.92\% \)
4. \[ P_v = (100,000)\]
\[ I = 10\%\]
\[ N = 360\]
\[ P_m = 877.57\]
\[ P_m = 877.57\]
\[ I = 10\%\]
\[ N = 240\]
\[ P_v = OLB = ? = 90,938\]
\[ P_v = OLB - 20,000 = 70,938\]
\[ I = 8\%\]
\[ N = 240\]
\[ P_m = 593.35\]
\[ I = 8\%\]
\[ N = 120\]
\[ P_m = 593.35\]
\[ P_v = OLB = ? = 48,905\]

\[ 877.57 \times 120 \text{ months} = 105,308 - (1000,000 - 90,938) = 96,246 \text{ interest expense on first ten years} \]

\[ 593.35 \times 240 \text{ months} = 142,404 - (90,938 - 20,000) = 71,466 \text{ interest expense on second loan} \]

\[ 96,246 + 71,466 = 167,712 = \text{total interest expense} \]

(A simpler way to see this is as follows: total monthly payments made = 105,308 + 142,404 = 247,712 + 20,000 lump sum principal was paid = 267,712 total payments to lender. Therefore the interest expense would be 267,712 − 100,000 = 167,712.)

If no pay down or refi:
\[ 877.57 \times 360 = 315,925 - 100,000 = 215,925 = \text{total interest expense} \]

Interest savings = 215,925 − 167,712 = 48,213
Problem 4-13
(a) Property value = $105,000
Principal = $84,000
Interest rate = 12.00%
Maturity = 30 years
Loan origination fee = $3,500

Lender will disburse $84,000.00 less the loan origination fee of $3,500.00 or $80,500.00

(b) Monthly payments would be:
$84,000 \times \text{(MLC, 12\%, 30 years)} = $864.03

The effective interest cost would be:
$864.03 \times \text{(MPVIFA, 7\%, 30 years)} = $80,500

Solving for the interest rate, we get 12.58%

(c) The annual percentage rate (APR) is the same as the interest rate in part (b) rounded to the nearest .125%. Therefore, the APR is 12.625%.
Note to Instructors: APRs are rounded to the nearest 1/8 of a percent.

(d) Assuming the loan payoff occurs after 5 years, determine the mortgage balance:
Mortgage balance = PV of 300 monthly payments of $864.03 discounted at 12.00%
Mortgage balance = $82,037.10

The effective interest cost would be:
$864.03 \times \text{(MPVIFA, 7\%, 5 years)} + \$82,037.10 \times \text{(MPVIF, 7\%, 5 years)} = $80,500

Solving for the interest rate, we get 13.15%.

The effective interest rate in this part is different from the APR because the loan origination fee is amortized over a much shorter period (5 years instead of 30 years).

(e) With a prepayment penalty of 2% on the outstanding loan balance of $82,037.10, the penalty would be $1,640.72.

The effective interest cost would be:
$864.03 \times \text{(MPVIFA, 7\%, 5 years)} + \$83,677.85 \times \text{(MPVIF, 7\%, 5 years)} = $80,500

Solving for the interest rate, we get 13.44%.

This rate is different from the APR because penalty points are not used in the calculation of the APR.
Problem 4-14
Points required to achieve a yield to 10% for the 25 year loan.

Monthly payments:
\[
\begin{align*}
$95,000 \times (\text{MLC, 9\%, 25 years}) &= \text{Monthly payment} \\
$95,000 \times (\text{MLC, 9\%, 25 years}) &= $797.24
\end{align*}
\]

PV of 300 payments of $797.24 discounted at 10% = $87,773.67

Subtracting $87,773.67 from $95,000.00, we get $7,226.33

The loan origination fee should be $7,226.33 if the loan is to be repaid after 25 years and the lender requires a 10% yield.

If the loan is expected to be repaid after 10 years, the loan balance at the end of 10 years must be determined:

\[
\begin{align*}
\text{Loan balance after 10 years} &= \text{Present value of 180 payments of } $797.24 \text{ discounted at } 9\%.
\end{align*}
\]

\[
\begin{align*}
\text{Loan balance after 10 years} &= 78,602.27
\end{align*}
\]

Discounting by the desired yield of 10%:

\[
\begin{align*}
\text{Present value} &= $797.24 \times (\text{MPVIFA, 10\%, 10 years}) + $78,602.27 \times (\text{MPVIF, 10\%, 10 years})
\end{align*}
\]

Present value = $89,364.04

Subtracting $89,364.04 from $95,000.00, we get $5,635.96.

The loan origination fee should be $5,635.96 if the loan is to be repaid after 10 years, and the lender requires a yield of 10%.

Problem 4-16
(a) In order to find which loan is the better choice after 15 years, the effective interest rate of each loan must be calculated.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Loan A</th>
<th>Loan B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$75,000</td>
<td>$75,000</td>
<td>$75,000</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>10.00%</td>
<td>11.00%</td>
</tr>
<tr>
<td>Term (years)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Points</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Payment</td>
<td>$658.18</td>
<td>$714.24</td>
</tr>
<tr>
<td>Loan Balance after 15 years</td>
<td>$61,248.42</td>
<td>$62,840.44</td>
</tr>
<tr>
<td>Loan Balance after 5 years</td>
<td>$72,430.74</td>
<td>$72,873.48</td>
</tr>
</tbody>
</table>

**Loan A**
$70,500.00 = $658.18 \times (\text{MPVIFA, 7\%, 180 months}) + $61,248.42 \times (\text{MPVIFA, 7\%, 180 months})

Effective interest rate = 10.85%

**Loan B**
$73,500.00 = $714.24 \times (\text{MPVIFA, 7\%, 180 months}) + $62,840.44 \times (\text{MPVIF, 7\%, 180 months})

Effective interest rate = 11.29%

Loan A is the better alternative if the loan is repaid after 15 years.

(b) This part is solved the same as (a) except using the assumption that the loan is repaid after 5 years.

**Loan A**
$70,500.00 = $658.18 \times (\text{MPVIFA, 7\%, 60 months}) + $72,430.74 \times (\text{MPVIF, 7\%, 60 months})

Effective interest rate = 11.61%

**Loan B**
$73,500.00 = $714.24 \times (\text{MPVIFA, 7\%, 60 months}) + $72,873.48 \times (\text{MPVIF, 7\%, 60 months})

Effective interest rate = 11.53%

Loan B is the better alternative if the loan is repaid after 5 years.
FIN 444 - Real Estate Finance
Homework Assignment
Due Date:

The Dowless Company has decided to buy and develop a tract of land. Dowless secures a $200,000, 20-year (monthly payments), 12 percent mortgage on the property from D & D Mortgage Company. D & D charges 1 discount point on this loan as well as a 2 percent prepayment penalty on early debt retirement. After 4 years, Dowless changes plans and decides to sell the property and develop another site. Determine the effective yield (borrowing cost) on this loan.
FIN 444- Real Estate Finance
Quiz

1. When finance fees (origination or discount points) are charged and loans are prepaid, effective yields will be higher the sooner the loan is prepaid.

   True or False?

2. The effective yield will always exceed the nominal (contract) rate on loans that are prepaid.

   True or False?

3. Calculate the effective yield on a $100,000 loan with monthly payments (30 years) and an eight percent interest rate. The lender will charge one loan discount point and one origination fee point.