Problem Set #4

ECON 7010, Prof. Jason DeBacker
Due Tuesday, October 20, 2:40 p.m.

NOTE: Feel free to work in groups on these problems. However, I would like each of you to turn in your own solutions.

1. Consider the optimization problem of a firm which produces output from capital and labor. Assume that the firm’s stock on capital is given and that labor is hired at a real wage $w$. Suppose the firm is a monopolist in the product market and faces a demand curve with constant elasticity: $q^d = p^{-\gamma}$. Let $\Pi(K)$ be the firm’s profit after it has optimally chosen labor given the demand curve. Solve for $\Pi(K)$.

2. Consider the optimization problem of the firm with quadratic adjustment costs. Suppose that profit as a function of capital, $K$ and a shock, $A$, is given by: $\Pi(A,K) = AK^\alpha$ with $\alpha < 1$. What is the dynamic programming problem for the firm? Show that the value function is NOT proportional to $K$.

Now consider the non-stochastic growth model presented in class as a starting point. Each problem below outlines a stochastic version of the growth model. For each, write down the appropriate functional equation. Identify the state and control variables. Write down the conditions for optimality. Intuitively, how do the key macroeconomic variables (output, consumption, investment, employment) respond to these shocks?

3. Suppose the production function is stochastic: $y = AF(k)$. Assume $A$ follows a first-order Markov process.

4. Suppose that household preferences are given by $u(c, n, \varepsilon)$ where $c$ is consumption, $n$ is labor supply, and $\varepsilon$ is a shock to preferences. Assume that $\varepsilon$ follows a first-order Markov process. What assumptions are you making about $u(\cdot)$? Also, modify the technology so that there is a labor input (recall the class notes on the non-stochastic growth model with labor).

5. Return to the original problem where utility is just $u(c)$ and technology is $y = f(k)$. Suppose that the accumulation technology is given by $k' = k(1 - \delta) + i\varepsilon$, where $\varepsilon$ follows a first-order Markov process. What do you think the correlation will be between consumption and investment in this economy?