**Macro I: Midterm Exam #2**

ECON 7010, Prof. Jason DeBacker
Tuesday, November 10, 2:40 p.m.

**Instructions:** Answer all three questions. Each is worth 100 points with partial credit as indicated. You have 90 minutes to compete the exam.

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**Question 1**

Consider the overlapping generations model with production. In particular, assume that all agents live for two periods, working in the first (supplying labor of the amount \( n \in [0, 1] \)) and consuming in the second. There is no population growth. Preferences are given by \( U(c_{t+1}, n_t) = u(c_{t+1}) - g(n_t) \) (where \( u(c) \) is increasing and concave and \( g(n) \) is increasing and convex, with \( u'(0) = \infty \) and \( g'(1) = \infty \)). Output is produced according to \( y_t = f(n_t) \). Assume that output cannot be stored.

(a) Find the competitive equilibrium of the economy assuming no fiat money. (20)

**ANSWER:** The equilibrium without money is autarky: \( n_t = 0, \forall t \)

(b) Solve the planner’s problem. Does the planners solution leave individuals in the economy better off than in part (a)? (20)

**ANSWER:** Since each generation is the same, we can write the planner’s problem as:

\[
\max_n u(f(n)) - g(n) = 0
\]  

The FOC is \( u'(f(n))f'(n) = g'(n) \). Thus the allocation chosen by the planner is the \( n \) that makes this equation hold with equality. We know individuals will be better off here than in autarky given their preferences described above.

(c) Now assume fiat money is introduced into the economy. Specify the objective function and the budget constraint of a representative agent. (15)

**ANSWER:** The objective function is \( \max_{c_{t+1}, n_t} u(c_{t+1}) - g(n_t) = \pi_t f(n_t) \) and the budget constraint is given by: \( \pi_{t+1} c_{t+1} = \pi_t f(n_t) \). Solving for \( c_{t+1} \) using the budget constraint and then substituting into the objective function, we can write the problem as: \( \max_{n} u \left( \frac{\pi_t}{\pi_{t+1}} f(n_t) \right) - g(n_t) \)

(d) Find the necessary condition(s) of individual optimization with money. Interpret. (15)

**ANSWER:** The FOC is: \( \frac{\pi_t}{\pi_{t+1}} f(n_t) u' \left( \frac{\pi_t}{\pi_{t+1}} f(n_t) \right) = g'(n_t) \). This condition says that the marginal utility of consumption times the return to an hour of work (in units of consumption) is equal to the marginal disutility of work at the optimum.

(e) Solve for the competitive equilibrium with money. (30)

**ANSWER:** Market clearing says that \( \pi_t y_t = M \) or \( \pi_t f(n_t) = M \). We can use this to solve for \( n_t = \frac{f^{-1}(M)}{\pi_t} \), which we will make use of below. Plugging the solution to the market clearing condition into the FOC, then rearranging to get period \( t \) variables on the LHS and period \( t + 1 \) variables on the RHS we get: \( f(n_{t+1}) u'(f(n_{t+1})) = f'(n_t) \pi_t g'(n_t) \). This is the difference equation for all equilibrium allocations. Note that there is a unique, nontrivial SS at \( u'(n^*) f'(n^*) = g'(n^*) \). To find equilibrium prices, you will use \( n_t \) to solve for \( \pi_t = \frac{M}{f'(n_t)} \) (this comes directly from the market clearing condition).

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**Question 2**

Consider the problem of who uses capital to produce output and faces a profit function of \( \pi(K) = K^\alpha \).

Assume the firm faces quadratic costs to adjusting its capital stock \( c(K', K) = \frac{\gamma}{2} (\frac{K'}{K})^2 K \) and a depreciation rate of \( \delta \). The price of new capital is \( p^c \) and firms discount future profits at rate \( \beta \).
(a) Formulate the dynamic programming problem of the firm. (20)

\[ V(K) = \max_{K'} K^\alpha - p^k I - \frac{1}{2} \left( \frac{I}{K} \right)^2 K + \beta V(K'). \]

(b) What is the necessary condition for optimization? Interpret. (30)

**ANSWER:** The FOC (taken with respect to \( I \) or \( K' \)) is given by: \( p^k + \gamma \frac{I}{K} = \beta V'(K') \). This FOC says that firms invest up until the marginal benefit (the RHS) equals the marginal cost (the LHS). The marginal benefit is given by “Marginal Q”, or the change in firm value for an additional dollar of investment.

(c) Tobin’s Q-theory suggests that marginal Q is a sufficient statistics for the determination of firm investment. Write the equation showing that the firm’s investment rate is a function of marginal Q. (20)

**ANSWER:** Using the FOC from above, we can solve for the investment rate as:

\[ \frac{I}{K} = \frac{\beta q - p^k}{\gamma} \]  

where \( q = \text{marginal Q} = V'(K') \)

(d) Suppose that you wanted to study the impact of demand uncertainty on firm investment. Reformulate the problem so that it can address these questions. Please be sure to identify the state variables, the control variables, and to write the dynamic programming problem of the firm. (30)

The states are \( K \) and the demand shock, \( A \). The control is \( K' \) (or, equivalently, \( I \)). The problem would be something like:

\[ V(A, K) = \max_{K'} \pi(A, K) - p^k I - c(K', K) + \beta E_{A'|A} V(A', K') \]  

**Question 3**

Consider the problem of a two-period lived consumer. The agent supplies labor in the first period of life and incurs a disutility from doing so. The agent consumes in both periods, and utility in each period is increasing and concave in consumption. Assume that lifetime utility is additively separable and that the agent discounts period 2 by a rate \( \beta \). The agent is endowed with assets \( A_1 \) at the beginning of period 1 and earns a wage rate \( w \) for each unit of labor supplied. The agent can save or borrow at a gross real interest rate of \( R \).

(a) What are the state variables in this problem? What are the control variables? (25)

**ANSWER:** State variables = \( A_1 \) and maybe \( w \) (you might consider it a parameter). Controls = \( A_2, c_1, c_2, n \).

(b) Formulate the problem of the agent (25)

**ANSWER:** The problem of the agent can be given as:

\[ \max_{c_1, c_2, n} u(c_1, n) + \beta v(c_2) \]

subject to: \( c_2 = R(A_1 + wn - c_1) \)

(c) What are the necessary conditions of the agent’s problem? (30)

**ANSWER:** One can substitute the budget constraint in and simplify the above problem so that it is a function of only two choice variables. In that case, we have the following two first order conditions:

with respect to \( c_1 \): \( u_1(c_1, n) = \beta R v'(c_2) \)  

with respect to \( n \): \( -u_2(c_1, n) = \beta Rw v'(c_2) \)
(d) Suppose the consumer faces a borrowing constraint, such that savings have to be non-negative. Using words, pictures, or equations, show how this would impact the solution to the consumer’s problem. (20)

ANSWER: The consumer gets all her financial resources in the first period (initial assets and labor income) and so will never be borrowing from the future (where she gets no additional financial resources). Thus the borrowing constraint will have no impact on the problem.