Problem Set #2 SOLUTIONS: Economic Efficiency and Optimal Taxation

ECON 6430, Prof. Jason DeBacker
Due Thursday, February 20, 2:40 p.m.

1. Assume that labor supply is given by $L^S = w$ and that labor demand is given by $L^D = 60 - w$.

(a) Find the measure of excess burden for a $2 per hour tax on wages employees receive.

ANSWER: First, find the pre-tax eq’m: $w = 30, L = 30$. Now, introduce the tax and solve for the equilibrium with tax: $w = 31, L = 29$. The $EB = \frac{1}{2}bh = \frac{1}{2} \Delta L\tau = \frac{1}{2} * (30 - 29)(2) = 1$.

(b) Now assume the tax goes from $2 to $4. Find the additional excess burden from this tax increase.

ANSWER: Repeat the above. The eq’m with tax here is: $w = 32, L = 28$. Thus $EB = \frac{1}{2}(30 - 28)(4) = 4$, so the increase in excess burden (i.e, the marginal excess burden) is $MEB = 4 - 1 = 3$.

(c) How do the two numbers you just found compare?

ANSWER: The EB with the higher tax is equal to exactly the square of the change in tax rates.

(d) Draw this market, the interventions (taxes), and point out the deadweight losses.

2. Consider an intertemporal choice problem with utility function

$$\max \{c_1, c_2\} \quad U(c_1, c_2) = c_1^{\frac{1}{2}} + \frac{1}{3} c_2^{\frac{1}{2}}$$

The individual is endowed with $I_1 = 50,000$ and $I_2 = 10,000$. The rate of return is $r = .075$.

(a) Set up and solve the individual’s optimization problem. Solve for the optimal level of savings/debt, and consumption in both periods.

ANSWER: Solving this problem yields: $c_1 = 47,148.05$ and $c_2 = 13,065.85$. Note that since $c_2 > I_2$ we know that the individual is a net saver.

(b) Graph the intertemporal budget constraint, denote maximum consumption levels, and the endowment point, and denote the optimal choice and indifference curve.

ANSWER: Draw your normal IC over consumption of two goods (goods in period 1 and goods in period 2), where the slope of the budget constraint is determined by the real interest rate.

(c) Suppose “savings” implies any capital investment, the return on which is taxed at rate .25. Suppose “debt” denotes a mortgage loan, the interest on which is tax deductible (at the same rate). Solve the post-tax optimal savings and consumption levels, plot the new budget constraint on the same graph, above, and denote the new optimal choice.

ANSWER: In this case, $c_1 = 47,317.78$ and $c_2 = 12,388.09$. Again, the consumer is a net saver, even with the tax.

(d) Discuss the resulting income and substitution effects associated with part c.

ANSWER: We know there are both income and substitution effects going on and that they go in opposite directions. Since savings decreases after the tax, we know that the substitution effect dominates in this case.

3. Suppose the demand for good $X$ can be represented by the following equation: $Q^d_x = 22 - \left(\frac{1}{4}\right)P_x$. Furthermore, suppose that the demand for good $Y$ can be represented by $Q^d_y = 50 - P_y$. 


The prices of both goods are equal to $10. Suppose that an ad valorem tax is placed on both goods. Good Y is taxed at a rate of 5%. To ensure that the inverse elasticity rule holds, what must be the rate at which good X is taxed? (Hint: Elasticity at a given price is found using the formula $\eta^D_i = -\frac{1}{S_i}(P_t/Q_t)$, where $S_i$ is the slope of the demand curve for good $i$, $Q_i$ is the quantity demanded of good $i$, and $P_t$ is the price of good $i$.)

**ANSWER:** We have to determine elasticities of the two goods: $\eta^D_X = 0.25 * (10/19.5) = 0.128$ and $\eta^D_Y = 1 * (10/40) = 0.25$. The efficiency requires $(\text{elasticity of } X) \times \text{(tax on } X) = (\text{elasticity of } Y) \times \text{(tax on } Y)$, which translates into $0.128 \times \text{(tax on } X) = 0.25 \times 5 = 1.25$, which means that $X$ should be taxed at 9.7%.

4. Describe the Inverse Elasticity Rule in words? Why not tax all goods at the same rate?

**ANSWER:** Say something about taxing all goods at the same rate means that you’ll be forcing consumption of highly elastic goods down too much relative the revenue gained. That is, you’ll change behavior of consumers more for these goods than for others for each dollar of revenue generated. This is not efficient.