Problem Set #5
ECON 4020, Prof. Debacker
Due Thursday, April 30, 5 p.m.

NOTE: You are encouraged to work on this problem set in a group of up to four members. When finished, each group should turn in one copy of the problem set directly to me or to the departmental office on the fifth floor of Brooks Hall. A completed problem set should list the names of the group members who worked on the assignment. As noted in the syllabus, no late assignments will be graded.

1. Chapter 16, Problems and Applications (4 points): #2, #4

2. Chapter 16, “Made up problem” - Fisher 2-period life cycle model (5 points): An individual (who is representative of all the other individuals in the economy) lives for two periods. Period 1 is young, Period 2 is old. Assume that the real interest rate on savings and borrowing is $r = 0.05$.

   (a) If income in the first period of life is $Y_1 = 10$ and income in the second period is $Y_2 = 15$, write down the intertemporal budget constraint.

   (b) If the lifetime utility function of the representative agent is $U(C_1, C_2) = (C_1 \cdot C_2)^{0.5}$, calculate the optimal levels of $C_1$ and $C_2$ given the intertemporal budget constraint from part (a). Is the agent a saver or a borrower?

   (c) What are the optimal levels of $C_1$ and $C_2$ if the agent is borrowing constrained (i.e., $S \geq 0$)?

   (d) What would be the effect on consumption $C_1$ and $C_2$ of a government policy that increased income in the second period of life to $Y_2 = 20$?

   (e) What would be the effect on consumption $C_1$ and $C_2$ of a government policy that increased income in the second period of life to $Y_2 = 20$ and implemented a loan program that allowed $S < 0$? That is, the government facilitated borrowing for all possible points on the new intertemporal budget constraint. Why does this answer differ from that of part (d)?

3. Chapter 17, “Made up problem” - Neoclassical Model of Investment (4 points): Assume and economy populated by two types of firms - production firms and rental firms. Production firms rent capital $K$ from rental firms at a nominal price $R$. Production firms also hire labor $L$ at a nominal wage $W$ in order to produce output $Y$ sold at a nominal price $P$. The production function of production firms takes the following Cobb-Douglas form:

   \[ Y = 0.5K^{0.5}L^{0.5} \]  

   (0.1)

   The key decision for production firms is how much capital $K$ to rent from rental firms. On the other hand, rental firms rent capital $K$ to production firms at the nominal price $R$. Their nominal costs are given by $P_KK(r + \delta)$, where $r$ is the real interest
rate and $\delta$ is the rate of depreciation. The key decision for rental firms is how much capital to buy at market price $P_K$.

(a) Derive the demand function for capital $K^D$ on the part of production firms that maximizes their profits. This will be a function of the real rental rate of capital. Show that $K^D$ is a negative function of the interest rate.

(b) Derive the function for the supply of capital $K^S$ (that rental firms supply to production firms) that maximizes the real profits of rental firms. This will be a function of the real price of capital, the real interest rate, and the depreciation rate. [NOTE: This involves substituting the $MPK$ relation from the production firms’ problem into the rental firms’ profit maximization problem.]

(c) Investment $I$ is a flow and capital $K$ is a stock, so $I = \Delta K + \delta K$. Show the channel of the effect on $I$ of the following exogenous shocks:
   - Anti-inflationary monetary policy raises the real interest rate.
   - An earthquake destroys part of the capital stock.
   - Immigration of foreign workers increases the size of the labor force.

(d) When the stock market crashes— as it did on October 1929, October 1987, and October 2008— how should the Federal Reserve respond? Why?

4. Chapter 18, Problems and Applications (6 points): #1, #3, #5

5. Chapter 19, Problems and Applications (6 points): #1, #2, #3