Chap 2 - The Budget Constraint

"The Budget Constraint (BC) tells us how much the consumer can afford.

e.g. if we walk into a pizza parlor and $10
    beer is $2, slices are $1,
    we could write the BC as:
    
    \[ 2 \times \text{beer} + 5 \times \text{pizza} = 10 \]

→ our consumption bundle is the combination of consumption goods chosen

→ e.g. consumption bundle is (4 beers, 3 slices)

→ we can write the BC w/ any number of goods,
    but usually focus on the case of

    2 goods

→ this helps us so that we can represent the problem in 2 dimensions

→ 2 goods are usually enough b/c

    we can think about one of the goods as a composite good

→ e.g. we consume beer and everything else - everything else is a composite good

→ when we have a composite good, we usually give it a price of $1 (so its dollars spent on everything else)
In general, with 2 goods, $x_1$ and $x_2$, we'll write the BC as:

$$p_1 x_1 + p_2 x_2 \leq m$$

- $x_1 = \text{quantity of good 1}$
- $x_2 = \text{quantity of good 2}$
- $p_1 = \text{price of good 1}$
- $p_2 = \text{price of good 2}$
- $m = \text{income}$

This equation says that the total amount spent on good 1 ($p_1 x_1$) plus the total amount spent on good 2 ($p_2 x_2$) cannot exceed total income ($m$).

We call the set of affordable consumption bundles (i.e., those that cost less than $m$) the consumers budget set.

The consumers budget line is the set of bundles that cost exactly $m$:

$$p_1 x_1 + p_2 x_2 = m$$
Graphically, we have:

Just do positive quadrant since consumption typically restricted to be non-negative

What's the slope of the budget line?

> Write the in slope-intercept form:

\[ P_1 x_1 + P_2 x_2 = m \]

\[ P_2 x_2 = m - P_1 x_1 \]

\[ x_2 = \frac{m}{P_2} - \frac{P_1}{P_2} x_1 \]

\[ y = a + b x \]

\[ a = \frac{m}{P_2} = \text{intercept} \]

\[ b = -\frac{P_1}{P_2} = \text{slope} \]
Another way to find the slope of the BL:

1. Consider a change in the amounts of goods 1 and 2 consumed that keeps one on the same BL:
   \( p_1 x_1 + p_2 x_2 = m \)
   and

2. \( p_1 (x_1 + dx_1) + p_2 (x_2 + dx_2) = m \)

- \( dx_1 \) and \( dx_2 \) are changes, but they can’t affect total spending.
- Now subtract 1 from 2:

\[
\begin{align*}
\frac{p_1}{p_1} x_1 + p_1 dx_1 + p_2 x_2 + p_2 dx_2 &= m \\
-p_1 x_1 + p_2 x_2 &= m \\
p_1 dx_1 + p_2 dx_2 &= 0
\end{align*}
\]

\( p_1 dx_1 + p_2 dx_2 = 0 \)

\( p_1 dx_1 = -p_2 dx_2 \)

\( \frac{dx_1}{dx_2} \) gives the rate at which good 2 can be substituted for good 1 while keeping total spending unchanged.

The slope of the BL gives the opportunity cost of consuming good 1 (i.e. how many units of good 2 need to be given up to consume another unit of good 1).
How the budget line changes

Changing income

$\Rightarrow$ Shifting the BL

- If income changes, this will shift the BL in or out.

- It will retain the same slope since $p_1$ and $p_2$ didn't change.

Consider $m \uparrow$ from $m$ to $m'$.

\[ \text{Slope} = \frac{-p_1}{p_2} \]

Changing prices:

$\Rightarrow$ Rotating the BL

- If prices change, this may affect the slope of the BL.

- Consider a $\uparrow$ in $p_1$ from $p_1$ to $p_1'$.

\[ \text{Slope} = \frac{-p_1'}{p_2} \]

In this case, the BL pivots out.
→ What happens if both prices change?
→ Consider case where $p_1$ and $p_2$
  go up by same amount
→ Say both + times as large

initially:
\[ p_1 x_1 + p_2 x_2 = m \]

after price change:
\[ p_1 x_1 + p_2 x_2 = m \]

\[ p_1 \bar{x}_1 + p_2 \bar{x}_2 = \frac{m}{t} \]
as if income became + times smaller

→ This is a shift in the BL,
  not a twist
  → slope still \(-\frac{p_1}{p_2}\) bc
  both prices changed
  by the same factor
Summary of price changes:

1) If one price changes, the BL pivots by the same amount.
2) If both prices change, the BL shifts parallel to original.
   - by different factors, the BL shifts and is not parallel to the original.

The Numerator

Notice that if one divides both sides of the BL equation by the same factor, nothing changes.

i.e. \[ P_1x_1 + P_2x_2 = m \]

\[ \frac{P_1}{P_2}x_1 + x_2 = m \]

Thus we can always make a normalization to our BL by setting a

1) one price equal to one:

\[ \frac{P_1x_1 + P_2x_2 = m}{P_2} \]

\[ \frac{P_1}{P_2}x_1 + x_2 = \frac{m}{P_2} \]

or
2) by setting income equal to one:

\[ P_1 x_1 + P_2 x_2 = m \]

\[ \frac{P_1 x_1}{m} + \frac{P_2 x_2}{m} = \frac{m}{m} \]

\[ \Rightarrow \frac{P_1 x_1}{m} + \frac{P_2 x_2}{m} = 1 \]

→ neither of these normalizations affect the

→ at all

→ we refer to the good whose price

→ we set equal to one as the

→ numeraire good

→ why so this?

→ It is often helpful to have a numeraire

→ goal as it means there will be one
good that one needs to solve
less price one needs to solve
for.
Representing taxes, subsidies, and revenues in a BC.

- Taxes
  - 3 types
    - Quantity tax - an amount per unit
      \[ \text{price} = p + \tau \]
    - ad valorem tax - a percentage of the sale price
      \[ \text{price} = (1 + \tau)p \]
    - excise tax - a sales tax

Diagram:

- BC pivot
- \( m \) axis
- \( BC \)
- \( x' \) axis
- \( m' \) axis
- \( y' \) axis
- \( x \) axis
- \( y \) axis

Example: gasoline tax, federal = 18.4 cents/gal.
3) A lump sum tax - same amount regardless of behavior (e.g., regardless of quantity purchased)

- it's yours

- there shall be BC b/c offset income.

\[ P_1 + P_2 = M - T \]

where \( T \) = lump sum tax amount

- Subsidies

- likely negative taxes

- maybe quantity, not volume, lump sum

- 1st defined, tax credit or energy efficient appliances

- Rationing

- limits on quantities consumed

- e.g., water during winter

- all other goods

[Diagram of budget set and tax impact]
**Kinked Budget Constraints**

Budget constraints may exhibit kinks if prices (before or after tax) change as quantities consumed change.

- A kink in a pound where the slope of the BC changes.

- E.g., 10% tax credit on energy efficient appliances only applies for cost up to $500.

\[
\begin{align*}
\text{Slope of the BC:} & \quad \text{slope} = -\frac{(1-0.1) Peef}{1} \\
\text{Slope of energy eff. appliances:} & \quad \text{slope} = -\frac{Peef}{1}
\end{align*}
\]

\[\frac{$500}{Peef}\]

→ Kinked BC's will introduce some complications to our solution of the consumer's problem.

→ We can still solve, just less straightforward.