Today- HH Problem:

- 2-period deterministic HH Problem
- 2-period HH Problem with stochastic income

Household optimization

- What is dynamic?
  - savings; state=wealth, control=consumption/future wealth
  - expenditures on durables; state=stock of durables, control=purchase of durables
  - human capital accumulation; state=education, control=continue in program/go to college
  - family size/structure; state=divorce/#adults/#kids, controls=?
  - health; state=health, control=exercise/smoke/health expenditures
  - employment status; state=employed or unemployed, control=search when unemployed

- We’ll look at savings first and derive some important macro results concerning the HH’s problem.
- We’ll also have our first look at how these theoretical models tie into empirical analysis.
- While our focus is on savings first, you should be able to see how these results generalize.

2-period Household Problem

- Non-stochastic case: \( \max(c_0) + \beta u(c_1) \), s.t. \( c_0 + \left( \frac{c_1}{R_0} \right) = y_0 + \left( \frac{y_1}{R_0} \right) + A_0 \)
  - endowment \( y_t \) in period \( t = 0, 1 \) \( \rightarrow \) labor income
  - endowment of \( A_0 \) from previous generation \( \rightarrow \) non labor income
  - \( R_0 \) is return on borrowing/lending
  - FOC: \( u'(c_0) = R_0 \beta u'(c_1) \)
  - draw graph with period 0 and period 1 consumption on each axis. Show that if \( \beta R_0 = 1 \) then indifference curve tangent at 45 degree line (because only way marginal utilities equal is if consumption in each period equal)

- Stochastic Income Case
  - \( y_0 \) known before choosing saving
  - \( y_1 \) not known until period 1
  - \( \max_{c_0} E_{y_1|y_0} \{ u(c_0) + \beta u(R_0(A_0 + y_0 - c_0) + y_1) \} \)
    - Show step were pass expectations through
    - FOC: \( u'(c_0) = \beta R_0 E_{y_1|y_0} u'(c_1) \)
* \( \beta R_0 = 1 \Rightarrow u'(c_0) = E_{y_1|y_0} u'(c_1) \) does not imply \( u'(c_0) = u'(c_1) \)

- **Example:** Highlight \( \frac{\partial c_0}{\partial y_0} \) (how does consumption vary as income varies)
  - \( u(c) = a + bc - \left( \frac{d}{2} \right) c^2 \) \((a, b, d)\) are parameters
  - first order process for \( y_t \)
    * \( y_t = \rho y_{t-1} + \varepsilon_t \), \( \rho \) is a parameter - it parameterizes the persistence of the income process
    * We assume \( E \varepsilon_t = 0 \), thus we know \( E(y_1) = E\{\rho y_0 + \varepsilon_1\} = E\rho y_0 + E\varepsilon_1 = \rho y_0 \)
  - \( \beta R_0 = 1 \) assumption
  - How rewrite \( u'(c_0) = \beta R_0 E_{y_1|y_0} u'(c_1) \) with the above assumptions?
    * \( b - dc_0 = E_{y_1|y_0} \{b - d(R_0(A + Y_0 - c_0) + y_1)\} \)
    * can solve this for \( c_0: c_0 = R_0(A + y_0 - c_0) + E_{y_1|y_0} y_1 \)
      - \( b/c \) with linear function we can pull the expectations operator through
        * \( \Rightarrow c_0 = \frac{R_0(A + y_0) + \rho y_0}{1 + \rho} \)
        * \( \Rightarrow \frac{\partial c_0}{\partial y_0} = \frac{R_0 + \rho}{(R_0+1)^2} > 0 \Rightarrow \rho \uparrow \Rightarrow \frac{\partial c_0}{\partial y_0} \uparrow \)
    - if \( \rho \) close to 1, means high earnings now imply high earning later - persistence. So consume more as \( y_0 \) increase because income increase is more permanent (if \( \rho = 1 \) consumption increases dollar for dollar with income because a permanent increase in income)
  - This is exactly Milton Friedman’s Permanent Income Hypothesis.
    * This theory sought to explain the “consumption puzzle”
      * The puzzle was the the Keynesian consumption function models could not explain the empirical fact that the average propensity to consume \( \frac{C}{Y} \) falls as income rises in the short run, but is flat as income rises when looking over longer time periods.
      * The PIH proposes that consumption responds more to permanent income changes than transitory changes. Thus you get a falling APC in the short run because consumption doesn’t change so much for transitory increases in income. But these transitory shocks average out in the long run - so in the long run, consumption is a function of permanent income.