A View of Economics (applies to basically all fields):

- Circle with three nodes:
  1. Theory
  2. Facts (data)
  3. Policy

- Each feeds to and from one another:
  - Theory
    - It is: Optimization, Equilibrium Analysis (i.e. a consistency requirement, how do all the individual decisions fit together?)
    - It is influenced by the facts: they shape theories we come up with
    - It influences Policy Design
  - Facts
    - It is: Statistics/Econometrics
    - It is influenced by Policy, which affects economic actions and outcomes measured in data
    - It influences Theory and Policy through evaluation of models and policy (i.e. test theory and policy efficiency with data)
  - Policy
    - It is: Evaluation and Design of economic policy
    - It is influenced by data via it’s evaluation with data
    - It influences Theory since Policy affects data from which we form theory

Facts:

1. Economics Fluctuate
   - Draw GDP since 1900 or so - upward trend with small movements, larger movement in 1929, 1981, 2008
   - Constant (or just about) growth over time (very close to the 3% per year average)
   - Business cycle fluctuations around this constant trend growth
   - Why?
     - shocks to technology, tastes/preferences (Real business cycle theory (RBC))
     - beliefs (i.e. animal spirits) (Keynesian business cycle theory)
     - natural cycle (Austrian business cycle theory)
   - very hard to know the timing of turning points in cycles
   - Macroeconomists not sure of the “whys”
     - Best modern models mix elements of RBC and Keynesian models
     - Modern macroeconomists focus on “micro-foundations”
   - What can (should) we do?
Leading economic models differ on both normative and positive points here.

Real business cycle models often suggested that we should not do anything to counter business cycles - that they represent economic agents making optimal decisions given changes in productivity.

It’s hard to believe that business cycles are optimal, and many, including neoclassical economists propose models where business cycles are not optimal.

However, many neoclassical models (since they rely on microeconomic behavior) can have a hard time supporting active policy making (i.e. they say we should do something, but that we cannot do something about business cycles).

2. Co-movement: Positive correlation between macroeconomic variables and Y (GDP)

- Notation
  - $Y =$ output (GDP)
  - $C =$ consumption
  - $I =$ Investment
  - $N =$ employment
  - $w =$ real wage
  - $r =$ real interest rate
  - all variables measured in real terms (not nominal)

- Correlations:
  - $\text{corr}(C, Y) > 0$
  - $\text{corr}(I, Y) > 0$
  - $\text{corr}(N, Y) > 0$
  - $\text{corr}(\frac{Y}{N}, Y) > 0$
  - $\text{corr}(w,Y) \approx 0$ - this and next are real challenge for model builders - it’s difficult to create a model with near zero elasticities in equilibrium
  - $\text{corr}(r,Y) \approx 0$

3. Standard Deviations

- $\text{std}(I) > \text{std}(Y) > \text{std}(C)$
- this is because of consumption smoothing (i.e., risk averse agents prefer to spread consumption across periods in an even manner)
- investment series is extremely volatile, consumption is less so (w/ durables more volatile than non-durables because durables are more like an investment)

4. Serial Correlation

- Positive serial correlations (Persistence $\rightarrow$ good yesterday, likely good today)
- $\text{corr}(x_{t}, x_{t-1}) > 0$
- $x$ could be $Y$, $C$, $I$, $N$, $w$, $r$...

5. There are the types of relationships that macroeconomic models hope to capture.

6. A test of how good the model is is how well it captures these (and other) “stylized facts”.

7. To reiterate - the challenge is to build up a model from individual optimization that captures the movements we see in the macroeconomy.

- There are big hurdles to doing this kind of economics - you need some “tools”.
- It is the learning of these tools that is the real goal of this course.
- In particular, we will learn dynamic optimization and general equilibrium modeling.
• These tools will serve you well outside of macroeconomics.

Cake Eating Problem:
• time, \( t = 1, 2 \)
• \( c_t \equiv \) consumption of cake in period \( t \)
• Preferences: \( u(c_1) + \beta u(c_2) \)
  - \( u'(\cdot) > 0 \)
  - \( u''(\cdot) < 0 \) (i.e., strictly concave utility function)
  - \( 0 \leq \beta \leq 1 \) discount factor
  - \( u'(0) = \infty \), Inada condition (first derivative approaches infinity as \( c \) approaches zero), always keeps you away from boundary conditions/corner solutions

• Endowment:
  - \( w_1 > 0 \) given (start of period one)
  - No endowment in period 2 (it’s important that agent knows this at outset)

• Technology:
  - Storage technology: \( w_2 = w_1 - c_1 \) (this is called the “transition equation”)
    * Storage technology is: “how much of that stuff that I put in today is there tomorrow”

• Markets:
  - None here

• Information:
  - No uncertainty

• The problem:
  - \( \max_{c_1, c_2, w_2, w_3} u(c_1) + \beta u(c_2) \)
    * subject to:
      - \( w_2 = w_1 - c_1 \)
      - \( w_3 = w_2 - c_2 \)
      - \( c_t \geq 0, t = 1, 2 \) - Inada condition takes care of this condition and ensures interior solution
      - \( w_t \geq 0, t = 2, 3 \)
      - Note that there will be 6 Lagrange multipliers for the 6 constraints
      - However, with some substitutions, we can eliminate some constraints
      - As noted, the Inada condition takes care of two constraints
      - Then one can combine the first two constraints into one: \( w_3 + c_1 + c_2 = w_1 \) and we’ll use \( \lambda \) as the Lagrangian multiplier on this constraint. Note this also gets rid of \( w_2 \) as a choice variable
      * Which leaves only one more constraint, \( w_3 \geq 0 \), we’ll use \( \phi \) as the Lagrangian multiplier on this constraint (only one more left since \( w_2 \geq 0 \) is implied by the two remaining constraints)
    - Lagrangian: \( L = \max_{c_1, c_2, w_3} u(c_1) + \beta u(c_2) + \lambda (w_1 - c_1 - c_2 - w_3) + \phi(w_3) \)
  - FOCs:
    * w.r.t. \( c_1 \): \( u'(c_1) = \lambda \)
    * w.r.t. \( c_2 \): \( \beta u'(c_2) = \lambda \)
Note that the two conditions above imply the “Euler” equation: \( u'(c_1) = \beta u'(c_2) \)

We’ll see these Euler equations all the time.

They relate two variables across time.

They are the condition of inter-temporal optimization.

This condition is necessary, but not sufficient condition for choices along an optimal path in a dynamic optimization problem.

Interpretation: If (discounted) marginal utilities are not equal, then agent can improve utility by rearranging the amounts consumed in different periods.

DRAW inter-temporal budget constraint and indifferent curve (whose slope is the ratio of marginal utilities).

\* w.r.t. \( w_3 \): \( \phi = \lambda \)

If \( \phi > 0 \), then that means the non-negativity constraint on \( w_3 \) binds, thus \( w_3 = 0 \).

We assumed that the marginal utility of consumption was positive (i.e., \( u'(c) > 0 \)), thus \( \lambda > 0 \) and so \( \phi > 0 \).

Thus we know that \( w_3 = 0 \) (i.e., we don’t leave any cake left over for period in which we get no utility from consuming it).

Since agents only receive an endowment in period 1 and get no utility from period 3 consumption, we can rewrite this problem in a more simple way:

\* \( c_1 + c_2 = w_1 \)

\* \( w_1 - c_1 = s \), where \( s = \text{savings} \)

\* \( c_2 = s \)

\* now the maximization problem becomes: \( \max_{s,w_1 \geq s \geq 0} u(w_1 - s) + \beta u(s) \)

\* the FOC (now just w.r.t. \( s \)) becomes the Euler equation: \( u'(w_1 - s) = \beta u'(s) \)

We can write the optimization problem as a Bellman equation: \( V_2 \equiv \max_s u(w_1 - s) + \beta u(s) \)

\* \( u'(w_1 - s) = \beta u'(s) \rightarrow \) how agent acts optimally is given by the Euler equation

\* \( s(w_1) \rightarrow c_1 \) and \( c_2 \) as a function of \( w_1 \)

\* This is the policy function or decision rule (demand function is a specific example of this)

\* describes how agents chose endogenous variables as a function of exogenous variables and parameters

\* \( V_2(w_1) = u(w_1 - s(w_1)) + \beta u(s(w_1)) \) (where \( V_2 \) is the value once I know how the agent will optimize (from policy function above))

We spend weeks extending this simple example - adding periods, changing the “storage technology”, adding uncertainty, etc. This will build up our dynamic optimization tools. We’ll then apply these tools to real economic questions.