Today:
- Models of economic fluctuations with money
- Models of economic fluctuations with asymmetric info

Azariadis, (1981):
- Assumes: \( u(c) = c; g(n) = \frac{1}{2}n^2 \)
- Subbing in the BC, we get: \( \max_{n_t} E_{P_{t+1}, X_{t+1} | s_t} \left( \frac{p_{n_t} X_{t+1}}{p_{t+1}} - \frac{1}{2}n_t^2 \right) \)
- The FOC is thus: \( E_{P_{t+1}, X_{t+1} | s_t} \left( \frac{p_{n_t} X_{t+1}}{p_{t+1}} \right) = n_t \)
- Using our functional form assumptions, we get: \( E_\Phi = \phi(M_\theta X') = \psi(z) \)
- Knowing that, in eq'n, \( \phi(z) = \frac{1}{\phi(z)} \cdot \psi(z) \cdot M_\theta X' = \psi(z) \)
  - Note that we get this from the market clearing conditions
- These functional form assumptions imply that the fundamental equation characterizing the SREE becomes: \( E_{(\theta', x', \theta | z)} \left( \frac{\theta', \psi(z')}{\theta} \right) = (\psi(z))^2, \forall z \)
  - Since iid, \( \theta \prod_{i=1}^{\theta', x'} \) iid so just calc their expectation from a known distribution
  - Note that \( k^2 \) is determined as part of the equilibrium and is not arbitrary (because \( \psi(\cdot) \) is an equilibrium function)
  - \( k^2 \) is what is called the “natural rate of output” - this is the long run level of output the economy tends to
    - e.g. if \( \theta = 1 \), \( \psi(z) = k = \) output
    - \( \Rightarrow k^2 \cdot (M(z))^2 = (\psi(z))^2, \forall z \)
    - \( \Rightarrow k \cdot M(z) = \psi(z), \forall z \rightarrow \) an equilibrium condition
- Define some functions:
  - \( (M(z))^2 = E_{\theta | z} \left( \frac{1}{\theta} \right) \)
  - \( E_{\theta, x}[\theta \psi(z)] = k^2 \Rightarrow E_{\theta, x}[\theta \cdot k \cdot M(z)] = k^2 \) or \( k = E_{\theta, x}[\theta \cdot M(z)] \)
  - Recall assumption (*) from previous lectures (that \( Pr(\theta \leq \theta | z) \) is increasing in \( z, \forall \theta \)), this assumption \( \Rightarrow M(z) \) is increasing in \( z \) (from \( k \cdot M(z) = \psi(z) \))
    - See this by: if \( z \uparrow, \psi(z) \uparrow \), but \( k \) is a constant, so for equality to hold \( M(z) \uparrow \)
- Examples:
  1. \( x \) random, \( \theta = 1 \)
    - If this, then we see that \( \psi(z) = k \) (i.e., output constant and independent of nominal shocks, \( x \))
2. $x = 1, \theta$ random
   - Try this as an exercise (PS 6, #1)
3. Noisy price signals
   - $\theta \in \{\theta_1, \theta_2\}, \theta_1 < \theta_2, Pr(\theta = \theta_1) = Pr(\theta = \theta_2) = \frac{1}{2}$
   - $x \in \{\theta_1, \theta_2\}$
   - $z \in \{\frac{\theta_1}{\theta_1}, 1, \frac{\theta_1}{\theta_2}\} = \{z_1, z_2, z_3\}$
   - $M(z_1)^2 = E_{\theta|z_1}(\frac{1}{\theta}) = \frac{1}{\theta_2} < \frac{1}{\theta_1}$
   - $M(z_2)^2 = E_{\theta|z_2}(\frac{1}{\theta}) = \frac{1}{\theta_1} + \frac{1}{\theta_2} < \frac{1}{\theta_1}$
   - $M(z_3)^2 = E_{\theta|z_3}(\frac{1}{\theta}) = \frac{1}{\theta_1}$
   - Note with the above that assumption (*) is satisfied b/c $M(z) \uparrow$ as $z \uparrow$
   - $x^2 \equiv \frac{1}{2}\theta_1\psi(\frac{\theta_1}{\theta_2}) + \theta_1\psi(\frac{\theta_1}{\theta_2}) + \theta_2\psi(\frac{\theta_1}{\theta_2}) + \theta_2\psi(\frac{\theta_1}{\theta_2}) = \frac{1}{4}\theta_1\psi(z_2) + \theta_1\psi(z_3) + \theta_2\psi(z_1) + \theta_2\psi(z_2)$
   - $\psi(z_1) = k \ast (\frac{1}{\theta_1})^{\frac{1}{2}}$, $\psi(z_2) = k \ast (\frac{1}{\theta_1} + \frac{1}{\theta_2})^{\frac{1}{2}}$
   - $\psi(z_3) = k \ast (\frac{1}{\theta_2})^{\frac{1}{2}}$
   - The 4 equations above will be used to solve for the equilibrium functions and constant:
     $\psi(z_1), \psi(z_2), \psi(z_3), k$

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Monetary Policy (Natural Rate Theory)

- Is anticipated monetary policy neutral? (Azariadis Section 3) (Note: Monetary policy = distrib of $x, f(x)$)
  - Natural rate theory says yes (these are the neoclassical guys who believe in the classical dichotomy - that nominal variables have no long run effect on real variables)
  - Azariadis says no
    - Take example 3 above with noisy prices and add a constant, $\omega$, to $x$
    - $x \in \{\theta_1 + \omega, \theta_2 + \omega\}$
    - $z \in \{\frac{\theta_1 + \omega}{\theta_1}, \frac{\theta_1 + \omega}{\theta_2}, \frac{\theta_2 + \omega}{\theta_1}, \frac{\theta_2 + \omega}{\theta_2}\}$
    - $E_{\theta}(\frac{1}{\theta})$ will change - in this case $z$ reveals $x$ and $\theta$
      - $\Rightarrow E(z) = (M(z))^2$, so $M(z)$ changes $\Rightarrow \psi(z)$ changes
    - “If I change $f(x)$, does $\psi(z)$ change?” - Yes, according to Azariadis, monetary policy matters

- To see that the above only works is add a constant, do the following exercise for the noisy price example:
  - Replace $x$ w/ $\tilde{x} = \lambda x$ ($\lambda > 0$, constant), then $\psi(z)$ is independent of $\lambda$
  - This is PS 6, #2

- What $f(x)$ is optimal?
  - Objective - what are you trying to do?
    - price stability
    - stabilize output
    - utility of representative agent (welfare)
      - lifetime utility $= E_{\theta, \omega}(u(\frac{\psi(z)}{\theta}) - g(\psi(z))) \equiv \tilde{W}$, where $\psi(z)$ is determined in equilibrium
      - pick $f(x)$ to maximize this... (i.e., $\max_{f(x)} \tilde{W}$)
  - Should we set $x = \tilde{x}$ w/ prob = 1?
    - This is what the natural rate guys (e.g. Lucas) thought
* Azariadis says no!
  - If you get rid of risk w x, you still have risk of θ
  - Getting rid of noise in x may not lessen uncertainty about z, so it is not welfare improving (b/c of incomplete markets - no way for agents to insure against population shock)
  - i.e., If x is a random variable, decreasing its variance may not help
  - May be ok with proper labor contracts (e.g., wage is promise to so many good next period, not today)