Problem Set #1 Solutions

ECON 6020, Prof. Jason DeBacker
Due Tuesday, January 26, 6 p.m.

NOTE: Feel free to work in groups on these problems. However, I would like each of you to turn in your own solutions.

Calculus review:

1. Find the derivative of \( y = f(x) = \ln(2x) \). Is this function concave or convex? How do you know?

   ANSWER: \( f'(x) = \frac{1}{2x} \cdot 2 = \frac{1}{x} \). \( f''(x) = -\frac{1}{x^2} \). Since \( f''(x) < 0 \), we know that the function \( f(x) \) is concave.

2. Let \( y = f(x_1, x_2) = x_2^2 + 2x_1x_2 + x_2 + 3 \). Find the first partial derivatives with respect to each variable, \( x_1 \) and \( x_2 \).

   ANSWER: \( \frac{\partial f(x_1, x_2)}{\partial x_1} = 2x_2 \). \( \frac{\partial f(x_1, x_2)}{\partial x_2} = 2x_2 + 2x_1 + 1 \).

3. Find the first and second derivatives of \( y = f(x) = (3 - x^2)^3 \). What rules do you need to use?

   ANSWER:

   \[
   f'(x) = 3(3-x^2)^2(-2x) \\
   = -6x(3-x^2)^2 \\
   = -6x(9 - 6x^2 + x^4) \\
   = -54x + 36x^3 - 6x^5
   \]

   \[
   f''(x) = -6x \cdot 2 \cdot (3-x^2) \cdot (-2x) + (-6) \cdot (3-x^2)^2 \\
   = 24x^2(3-x^2) - 6(3-x^2)^2 \\
   = 24x^2(3-x^2) - 6(9-6x^2 + x^4) \\
   = 72x^2 - 24x^4 - 54 + 36x^2 - 6x^4 \\
   = 108x^2 - 30x^4 - 54
   \]

   To solve for these derivatives, you need to use the chain rule and product rule and power rule.

4. Find the maximum of \( y = f(x) = -3x^2 + 2x + 5 \).

   To find the max, we need to find where the slope is equal to zero. The first derivative of the function is: \( f'(x) = -6x + 2 \). Thus we solve: \(-6x + 2 = 0\) for \( x \). Thus we find \( x = \frac{1}{3} \) is the point at which the slope is equal to zero. Thus this point on the function represents a max or min. To confirm that this is the maximum of the function, we find the second derivative. We find that \( f''(x) = -6 < 0 \). Thus this point does represent a maximum of the function since at \( x = \frac{1}{3} \), the slope is zero and decreasing.
5. Suppose that demand for apartments (where \( x \) is the quantity of apartments demanded and \( p \) the price of apartments) can be given by \( p = 600 - 50x \). Plot this equation on a graph that has \( p \) on the vertical axis and \( x \) on the horizontal axis.

Draw a straight line with intercept of 600 and a slope of -50.

6. Now let the long run supply of apartments be given by the equation \( p = 100x \). Plot this equation on the same graph as you drew for the previous question.

Now you’ll add to your graph a straight line that goes through the origin and has a slope of 100.

7. Solve for supply and demand in terms of quantities. That is, solve each equation so it gives \( x \) as a function of \( p \) (e.g., \( x = ap + b \)).

For demand, we have \( x = 12 - \frac{p}{50} \). For supply, we have \( x = \frac{p}{100} \).

8. Solve for the equilibrium. That is, solve for the \( x \) and \( p \) where supply equals demand.

To solve for these two unknowns, we’ll use the two equations - one for supply, one for demand. In equilibrium, supply = demand, so we have \( 600 - 50x = 100x \) (or \( 12 - \frac{p}{50} = \frac{p}{100} \)). Solving the former for \( x \), we get \( x = \frac{600}{150} = 4 \) (or solving the latter, we get \( \frac{3p}{100} = 12 \implies p = 400 \)). Plugging \( x = 4 \) into the equation for supply or demand, we get \( p = 600 - 50x = 600 - 50(4) = 600 - 200 = 400 \). So the equilibrium price and quantities are 400 and 4, respectively.

9. Look at your graph, does the analytical result make sense?

Looking at the graph you drew, you should see that the intersection of the two curves is at \( p = 400 \) and \( x = 4 \).

10. Now let’s consider a comparative static. How does the equilibrium price change if landlords pay a tax of $100 per unit they rent. Such that the new supply curve can be represented by \( p = 100x + 100 \)?

We do the same as above, but with the new supply curve and same demand curve. In the new equilibrium, we have supply equal demand, or \( 100x + 100 = 600 - 50x \). Solving for \( x \) we get \( 150x = 500 \implies x = \frac{500}{150} = \frac{10}{3} \approx 3.34 \). Putting this into the demand equation, we find \( p = 600 - 50x = 600 - 50(3.34) = 600 - \frac{500}{3} \approx \frac{1800 - 500}{3} = \frac{1300}{3} \approx 433.34 \). Thus, the tax reduces the number of apartments rented from 4 to 3.34 and increases their after-tax price from $400 to $433.34 (note that the after-tax rent the landlords receive is $433.34-$50=$383.34 - we’ll talk more about this later).