1. Chapter 16, Problems and Applications (10 points): #2, #4

   • #2
     (a) We can use Jill’s intertemporal budget constraint to solve for the interest rate:

     \[
     C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}
     \]

     \[
     100 + \frac{100}{1 + r} = 0 + \frac{210}{1 + r}
     \]

     The interest rate that makes this budget constraint hold is \( r = 10\% \). Jill borrowed $100 for consumption in the first period and used her second period income to pay $110 on the loan ($100 in principle and $10 in interest) and $100 for consumption.

     (b) The rise in interest rates leads Jack to consume less today and more tomorrow. This is because of the substitution effect: it costs him more to consume today than tomorrow, because of the higher opportunity cost of forgone interest. By the principle of revealed preference, we know that Jack is better off: at the new interest rate he could still consume his $100 in each period, so the only reason he would change his consumption pattern is if the change makes him better off.

     (c) Jill will consume less in the first period. She faces both an income effect and a substitution effect. Because consumption today is more expensive, she substitutes out of it. Also, since all her income comes in the second period, the higher interest rate raises her cost of borrowing, and thus lowers her income. Assuming consumption in period one is a normal good, this provides an additional incentive to lower consumption in period one and two. We know that Jill is worse off with the higher interest rates because her new budget constraint is entirely below her old budget constraint.

   • #4
     (a) Temporary fiscal policy is more potent when borrowing constraints are present. This is because borrowing constraints make it more likely that people would like to consume more today, but can’t because they can’t borrow
against future earnings. Recall that without borrowing constraints, Ricardian Equivalence suggests that a temporary tax cut will have no effect on consumption/aggregate demand.

(b) Future fiscal policy is less potent when borrowing constraints are present. This is because borrowing constraints make it more difficult to move that future increase in disposable income to today because people may already be borrowing all that they can.

2. Chapter 16, “Made up problem”- Fisher 2-period life cycle model (10 points):

(a) The intertemporal budget constraint is given by:

\[ C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r} \]

\[ C_1 + \frac{C_2}{1.05} = 10 + \frac{15}{1.05} \]

(b) Given that \( U(C_1, C_2) = \frac{C_1^{1/2} C_2^{1/2}}{C_1^{1/2} C_2^{1/2}} \), we can solve for the optimal choices of \( C_1 \) and \( C_2 \) by using the condition that the slope of the budget line and the slope of the indifference curve be equal at the optimal choice of \( C_1 \) and \( C_2 \) and the budget constraint.

- Step 1: Find the MRS and set it equal to the slope of the budget line to solve for \( C_2 \) in terms of \( C_1 \) and \( r \):

\[ MRS = \frac{MU_{C_1}}{MU_{C_2}} = \frac{\frac{1}{2}C_1^{-1/2}C_2^{1/2}}{\frac{1}{2}C_1^{1/2}C_2^{-1/2}} = \frac{C_2}{C_1} \]

The slope of the budget line equals the “price of \( C_1 \)” divided by the “price of \( C_2 \).” Looking at the budget constraint reveals that this is given by \( \frac{1}{1 + r} = 1 + r \). Therefore, setting the slope of the budget line equal to the MRS we get: \( \frac{C_2}{C_1} = 1 + r \), which means that \( C_2 = C_1(1 + r) \).

- Step 2: Plug the result of Step 1 (\( C_2 = C_1(1 + r) \)) into the budget constraint:

\[ C_1 + \frac{C_1(1 + r)}{1 + r} = Y_1 + \frac{Y_2}{1 + r} \]

\[ C_1 + \frac{C_1(1 + r)}{1 + r} = Y_1 + \frac{Y_2}{1 + r} \]

\[ C_1 + C_1 = Y_1 + \frac{Y_2}{1 + r} \implies C_1 = \frac{Y_1 + \frac{Y_2}{1 + r}}{2} \implies C_1 \approx 12.14 \]

- Step 3: Plug the result of Step 2 into the result from Step 1:

\[ C_2 = C_1(1 + r) = 12.14 \times 1.05 = 12.75 \]

Since \( C_1 > 10 \), the agent is a borrower.

(c) If borrowing is not allowed (i.e. \( S \geq 0 \)), then the optimal choices become \( C_1 = 10 \) and \( C_2 = 15 \).
(d) With the borrowing constraint, raising income in period 2 has no effect on consumption in period 1, but raises $C_2$ to 20.

(e) In this case, follow Steps 2 and 3 from part (b) above to find that $C_1$ becomes approximately 14.52 and $C_2$ becomes 15.25.

3. Chapter 17, “Made up problem”- Neoclassical Model of Investment (10 points):

(a) Production firms choose how much capital to rent $K^D$ by maximizing profits

$$\max_K PF(K, L) - RK - WL$$

The first order condition states that at the optimal choice of $K$:

$$\frac{\partial \Pi}{\partial K} = P * \frac{\partial F}{\partial K} - R = 0$$

Recall that $\frac{\partial F}{\partial K}$ is the marginal product of capital ($MPK$). Thus, the choice of $K$ must satisfy: $MPK = \frac{R}{P}$. Here, the marginal product of capital is $\frac{1}{4}(\frac{L}{K})^{\frac{1}{2}}$. Thus we can find the choice of $K$ as a function of the real rental rate:

$$K^D = \frac{L}{(4\frac{R}{P})^{\frac{1}{2}}}$$

Since $\frac{R}{P}$ is in the denominator, a higher real rental rate means less capital demanded.

(b) Rental firms choose how much capital to supply, $K^S$ in order to maximize their nominal profits:

$$\max_K (R * K) - P_K(r + \delta)$$

The first order condition states that at the optimal choice of $K$:

$$\frac{\partial \Pi}{\partial K} = R - P_K(r + \delta) = 0$$

There is no $K$ in this equation, but we know from solving the problem of producing firms that, $MPK = \frac{R}{P}$. Divide the above equation by $P$ to get (alternatively, you can get here by setting up the problem as maximizing real profits):

$$\frac{R}{P} = \frac{P_K}{P}(r + \delta) \implies MPK = \frac{P_K}{P}(r + \delta) \implies \frac{1}{4}(\frac{L}{K})^{\frac{1}{2}} = \frac{P_K}{P}(r + \delta)$$

Now we have $K$ in the equation and we can solve for $K^S$ by rearranging the equation so that $K$ is on a side all by itself. After some algebra we find:

$$K^S = \frac{L}{(\frac{4P}{K}(r + \delta))^2}$$

(c) Here you need to evaluate how these events affect the supply and demand for capital.

- Anti-inflationary policy $\implies r \uparrow \implies$ a higher cost of capital $\implies$ less capital supplied and lower investment
• An earthquake destroying capital means a higher $MPK$, which means more demand for capital and more investment
• Immigration $\implies L \uparrow \implies MPK \uparrow \implies$ more demand for capital and more investment

(d) The stocks prices represent the expected, discounted present value of all future dividends. A fall in stock prices means a decline in expected future profits. Since investment rates and profits are correlated, the stock market crashed signal a fall in future investment. To increase investment, the Federal Reserve can increase the money supply to lower the real interest rate. Lowering the real interest rate lowers the cost of capital and thus increases investment.