A note on chromatic number and odd induced cycles

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Abstract

An odd hole is an induced odd cycle of length at least 5. Scott and Seymour confirmed a conjecture of Gyárfás and proved that if a graph $G$ has no odd holes then $\chi(G) \leq 2^{\omega(G)+2}$. Chudnovsky, Robertson, Seymour and Thomas showed that if $G$ has neither $K_4$ nor odd holes then $\chi(G) \leq 4$. In this note, we show that if a graph $G$ has neither triangles nor quadrilaterals, and has no odd holes of length at least 7, then $\chi(G) \leq 4$ and $\chi(G) \leq 3$ if $G$ has radius at most 3. We also show that, for each vertex $u$ of $G$, the set of vertices of the same distance to $u$ induces a bipartite subgraph. This answers some questions by Plummer and Zha. This is joint work with Baogang Xu and Gexin Yu.