Given a cellular embedding of a graph $G$ into a surface $S$, local “combinatorial curvatures” are defined for each vertex or face, with curvature values dependent solely on the topology of the embedding. For such an embedding into such a bounded (compact) surface $S$ without boundaries, it turns out that the sum over these combinatorial curvatures either for the vertices or for the faces gives $2\pi$ times the Euler-Poincare characteristic for $S$. Notably, this is the same result as for Descartes’ defects summed over the corners of $S$ when $S$ is a piecewise planar (polyhedral-like) surface. Also this result is the same as for the Gaussian curvature integrated over $S$ when $S$ is a boundariless smooth (compact) surface $S$.

Moreover, this combinatorial curvature (for graph embeddings) seems to manifest further characteristics justifying its naming. Most generally it is speculated that for “reasonable” embeddings of $G$ into a surface $S$ which is in turn smoothly embedded in Euclidean 3-space $\mathbb{E}_3$, there should be a local matching between combinatorial & Gaussian curvatures. Here “reasonable” means that the edges of $G$ as occur in the overall embedding are of similar lengths $L$, and that the principal radii of curvature for the Gaussian curvature of $S$ are uniformly greater than this length $L$. Finally this matching speculation is suggested to be relevant in understanding conjugated-carbon nano-structures.